Pariksha Saathi — Quantitative Aptitude Notes

26 topics • 62 lessons • updated 28 October 2026

These notes are written by an AI assistant - they are the same lessons the Pariksha Saathi app shows. They have not been reviewed by a subject-matter expert. Check facts that matter against an official source or a standard textbook, and tell us about any error: parikshasaathi.com/contact/

The Quantitative Aptitude topics in the syllabus of: SSC CGL, IBPS PO, SSC MTS, SSC CHSL, SBI PO, SBI Clerk, IBPS Clerk, RRB NTPC, RRB Group D, RRB ALP, SSC GD Constable, Agniveer (Army GD), UP Police Constable, Bihar Police Constable, MP Police Constable, Rajasthan Police Constable, UPPSC Prelims, MPPSC Prelims, UPSSSC PET, BSSC Inter Level, Rajasthan Patwari, Haryana CET (Group C), DSSSB TGT. Each topic ends with a link to its free practice questions; full mock tests are free in the Pariksha Saathi app and on the website.

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Average

Average — Concepts and Applications

An arithmetic average equals the sum of observations divided by their count, so the original total can always be recovered as average × count. A weighted average is the sum of each value multiplied by its weight, divided by the total weight. Combine groups by adding their totals and counts rather than averaging group means without regard to group size. When observations are added, removed or replaced, adjust the total and count separately before finding the new average. A missing value is the required total minus the sum of known values, and correcting an erroneous entry changes the total by correct value minus recorded value.

A very common question type replaces ONE member of a group with someone new and reports how the average changes: if replacing a person changes the average of n people by d (up or down), the total changes by n×d, and since only one person was swapped, the new person's value is simply the old person's value ± (n×d) - no need to recompute the whole total from scratch.

For a set of consecutive integers (or any evenly-spaced sequence), the average is always exactly the midpoint of the first and last terms: average = (first + last)/2. When the count is odd, this average is also exactly the middle term of the sequence - a fast way to answer "find the average of these consecutive numbers" without summing anything.

Examples

- Find the arithmetic average of 11, 15, 18, 22, 29. Worked solution: The total is 95 and there are 5 observations. Average = 95/5 = 19. Check by centring the five values around 19: their deviations sum to 0, confirming the mean.

- 9 values average 26. Adding one more value raises the average to 27. Find the added value. Worked solution: The old total is 234. The new total is (9 + 1) × 27 = 270. Subtracting gives the added value 36. Check: the added value is 10 above the old mean, increasing the average of all 10 values by exactly 1.
- Four equally weighted tests each have a maximum of 100 marks. Scores on the first three are 63, 67 and 73. What fourth score produces an average of 71? Worked solution: The required total is 4 × 71 = 284; the first three scores total 203. Thus the fourth score must be 81. Check: the first three scores are collectively 10 below three target scores, so the last score must be 10 above 71, still within 100.
- The average weight of 8 people increases by 2.5 kg when a new person replaces one who weighs 65 kg. Find the new person's weight. Worked solution: The total weight increases by 8 × 2.5 = 20 kg (since only the count stays fixed at 8, and the average rose by 2.5 for all 8). The new person's weight = 65 + 20 = 85 kg.
- Find the average of 11 consecutive integers starting from 5. Worked solution: The sequence runs from 5 to 15. Average = (first + last)/2 = (5+15)/2 = 10 - which is also exactly the 6th (middle) term of the 11 numbers, as expected for an odd count.
- The average age of a class of 30 students is 15 years. If the teacher's age (40 years) is included, find the new average. Worked solution: Old total = 30 × 15 = 450. New total = 450 + 40 = 490, over 31 people. New average = 490/31 ≈ 15.8 years.
- Two sections of a class have 20 and 30 students with average marks 60 and 70 respectively. Find the combined average. Worked solution: Combine totals, not the two averages directly: total = 20×60 + 30×70 = 1200+2100 = 3300, over 50 students, giving a combined average of 3300/50 = 66 - not the simple midpoint 65, because the larger section pulls the average toward its own mean.

> **Quick revision:** Average = total/count; weighted average = Σ(value × weight)/Σweights; use totals before combining, adding, removing or replacing values. Replacing one member: new value = old value ± (count × change in average). Consecutive/evenly-spaced numbers: average = (first+last)/2, which equals the middle term when the count is odd. Combining groups: add totals and counts, never average the averages directly unless the groups are equal size.

Average Solved Questions

Average questions are mostly about handling the formula backwards — getting from an average back to a sum is more often the actual skill being tested than the plain average itself. This page covers the core formula, fast shortcuts for consecutive numbers and numbers close to a round base, how to combine group averages correctly, and several fully worked examples. Average = Sum of all values / Number of values. Reversed, this gives Sum = Average × Number of values — this reversal is the key to most average questions, which usually give you an average and ask for something about the sum (or vice versa). When a new value joins a group, work out the new total sum first (old sum + new value), then divide by the new count — never average the old average directly with the new value. Useful shortcut: if adding one new value to n existing values changes the average from A to A′, the new value itself = A′ + n(A′ − A). For a set of consecutive numbers (or any evenly spaced sequence), the average equals the middle value (odd count) or the average of the two middle values (even count) — equivalently, (first + last) / 2. This is faster than adding every term. Combining two groups: the combined average is the weighted average by group size, $(sum_1 + sum_2) / (count_1 + count_2)$ — not the simple average of the two group averages, unless the two groups happen to be the same size. Deviation method for quick mental averaging: pick a convenient round base near the numbers, find how much each number deviates from it (plus or minus), average those small deviations, then add the result back to the base. Sanity check: the average of a set must lie between its minimum and maximum value. If a computed average falls outside that range, there is a calculation error somewhere.

Examples

- Find the average of 12, 15, 18, 21, and 24. Solution: Sum = 12 + 15 + 18 + 21 + 24 = 90 Average = Sum / Count = 90 / 5 = 18 Answer: 18
- The average of 5 numbers is 27. If one number is excluded, the average of the remaining 4 numbers becomes 25. Find the excluded number. Solution: Sum of all 5 numbers = 5 × 27 = 135 Sum of the remaining 4 numbers = 4 × 25 = 100 Excluded number = 135 − 100 = 35 Answer: 35

- The average weight of 30 students in a class is 42 kg. When the teacher's weight is included, the average rises by 1 kg. Find the teacher's weight. Solution: Old sum (30 students) = 30 × 42 = 1,260 kg New average (31 people) = 42 + 1 = 43 kg, so new sum = 31 × 43 = 1,333 kg Teacher's weight = new sum − old sum = 1,333 − 1,260 = 73 kg Answer: 73 kg
- In a class of 40 students, the average marks are 60. There are 20 boys and 20 girls in the class. If the boys' average marks are 50, find the girls' average marks. Solution: Total marks (40 students) = 40 × 60 = 2,400 Boys' total marks (20 boys) = 20 × 50 = 1,000 Girls' total marks = 2,400 − 1,000 = 1,400, so girls' average = 1,400 / 20 = 70 Answer: 70
- Find the average of all natural numbers from 15 to 25, inclusive. Solution: This is a consecutive (evenly spaced) sequence, so average = (first + last) / 2. = (15 + 25) / 2 = 20 Answer: 20
- Section A has 20 students with an average score of 60, and Section B has 30 students with an average score of 70. Find the combined average for both sections. Solution: The section sizes differ, so the combined average is NOT the simple average (60+70)/2 = 65 — it must be weighted by the number of students. Total marks = (20 × 60) + (30 × 70) = 1,200 + 2,100 = 3,300 Total students = 20 + 30 = 50 Combined average = 3,300 / 50 = 66 Answer: 66
- Find the average of 198, 203, 207, 196, and 201 using the deviation method. Solution: Pick base = 200. Deviations from 200: −2, +3, +7, −4, +1. Sum of deviations = −2 + 3 + 7 − 4 + 1 = 5. Average deviation = 5 / 5 = 1. Average = base + average deviation = 200 + 1 = 201. Check by direct addition: 198+203+207+196+201 = 1,005; 1,005/5 = 201. Matches. Answer: 201
- Practice: Find the average of 8, 12, 16, 20, and 24. (Answer: 16. Sum = 80; 80/5.)
- Practice: The average age of 6 people in a room is 30 years. A seventh person enters and the average becomes 32 years. Find the age of the seventh person. (Answer: 44 years. New value = A′ + n(A′ − A) = 32 + 6(32 − 30).)
- Practice: A cricketer has an average of 40 runs over 10 innings. How many runs must he score in the 11th innings to raise his average to 42? (Answer: 62 runs. New sum needed = 11×42 = 462; old sum = 10×40 = 400; difference = 62.)
- Practice: Find the average of all integers from 21 to 29, inclusive. (Answer: 25. (first + last)/2 = (21+29)/2.)
- Practice: Section X has 15 students with an average score of 80, and Section Y has 25 students with an average score of 68. Find the combined average. (Answer: 72.5. Total marks = 15×80 + 25×68 = 2,900; total students = 40; 2,900/40.)
- Practice: Find the average of 505, 498, 510, 497, and 500 using the deviation method (base 500). (Answer: 502. Deviations: +5, −2, +10, −3, 0; sum = 10; average deviation = 2; 500 + 2.)
- Practice: The average marks of a class of 35 students is 72. The average of the first 20 students is 75. Find the average of the remaining 15 students. (Answer: 68. Total = 35×72 = 2,520; first 20 total = 20×75 = 1,500; remaining total = 1,020; 1,020/15.)

> **Quick revision:** Average = Sum of all values / Number of values; Reversed, this gives Sum = Average × Number of values — this reversal is the key to most average questions, which usually give you an average and ask for something about the sum (or vice versa); When a new value joins a group, work out the new total sum first (old sum + new value), then divide by the new count — never average the old average directly with the new value; Useful shortcut: if adding one new value to n existing values changes the average from A to A′, the new value itself = A′ + n(A′ − A); For a set of consecutive numbers (or any evenly spaced sequence), the average equals the middle value (odd count) or the average of the two middle values (even count) — equivalently, (first + last) / 2. This is faster than adding every term; Combining two groups: the combined average is the weighted average by group size, $(sum_1 + sum_2) / (count_1 + count_2)$ — not the simple average of the two group averages, unless the two groups happen to be the same size; Deviation method for quick mental averaging: pick a convenient round base near the numbers, find how much each number deviates from it (plus or minus), average those small deviations, then add the result back to the base; Sanity check: the average of a set must lie between its minimum and maximum value. If a computed average falls outside that range, there is a calculation error somewhere

Free practice questions: parikshasaathi.com/q/average-applications/

Data Interpretation

Data Interpretation — Concepts and Applications

Read the stated labels, units, periods and category definitions before calculating, and use only values supplied in the question. Totals add compatible quantities, differences subtract them, ratios compare them in the stated order and

percentages divide by the specified reference base. A combined average uses total quantity divided by total count, or weighted values when frequencies differ. A pie sector represents angle/360 of the total, and an ordinary bar chart's labelled heights represent the stated values. For histograms identify whether height is frequency or frequency density, use interval boundaries consistently, and for a frequency polygon use the supplied midpoint-frequency coordinates and treat midpoint-based means as estimates for grouped data.

For a pie chart specifically, converting a sector's angle into an actual value is a direct proportion: value = (sector angle/360°) × total value - the whole pie's 360° always represents the grand total, so any slice is that same fraction of the total.

When comparing two categories from a table, express the comparison exactly as asked: a RATIO keeps both quantities side by side in simplest form (e.g., 40:25 simplifies to 8:5), while a PERCENTAGE CHANGE from an old value to a new one is (new − old)/old × 100 - always dividing by the OLD (starting) value, not the new one, unless the question explicitly asks for the reverse.

Examples

- Table of notebooks sold: Shop A = 21, Shop B = 32, Shop C = 43. Find the total notebooks sold. Worked solution: Add the three entries: 21 + 32 + 43 = 96 notebooks.
- A table shows daily deliveries: Monday 5, Tuesday 8, Wednesday 14. Find the average deliveries per day over these three days. Worked solution: Total deliveries = 27 across 3 days. The average is 27/3 = 9.
- A table lists items sold by three shops: A = 36, B = unknown, C = 28. The total is 111. Find B's sales. Worked solution: The known entries total 36 + 28 = 64. Subtract from the overall total: B = 111 − 64 = 47.
- A pie chart shows a company's total monthly expenses of ₹50,000, with the "Transport" sector occupying 72° of the 360° circle. Find the amount spent on Transport. Worked solution: Transport = (72/360) × 50000 = 0.2 × 50000 = ₹10,000.
- Shop A sold 40 units and Shop B sold 25 units in a week. Find the ratio of A's sales to B's sales in simplest form. Worked solution: Ratio = 40:25. Dividing both by their HCF (5) gives 8:5.
- A product's sales rose from 250 units to 300 units between two months. Find the percentage increase. Worked solution: Percentage change = (300−250)/250 × 100 = 50/250 × 100 = 20% - note the OLD value (250) is the base, not the new value (300).

> **Quick revision:** Totals add, differences subtract, ratios compare in the stated order, percentages divide by the specified base. Combined average = total quantity/total count (weighted when frequencies differ). Pie sector value = (sector angle/360°) × total. Ratio: simplify by the HCF. Percentage change = (new − old)/old × 100 - always divide by the OLD value unless told otherwise.

Data Interpretation Solved Questions

Data Interpretation is mostly ordinary arithmetic wearing a table or chart as a disguise — reading the data accurately is the only genuinely new skill; the calculations themselves are Percentages, Ratios and Averages you already know. This page covers that core idea, two-series (sales-and-cost) tables, filling in a missing value from a given total, and weighted averages across categories, with several fully worked examples. Always check the units and any scale given with the table or chart ("figures in thousands," "sales in ₹ lakhs") before calculating — misreading the scale is the single most common careless mistake in this topic. For "what percentage more/less" questions, calculate the percentage change relative to the ORIGINAL value (the thing being compared against), not the new value — the same base-of-comparison rule from Percentages, just inside a table. For multi-step questions, compute each individual value needed FIRST and write it down, rather than trying to combine several lookups in your head at once. A ratio or an average computed from table data works exactly like the standalone Ratio/Average topics — the table just supplies the raw numbers. When a table mixes two related series for the same categories (e.g. sales AND cost, or income AND expenditure), a "profit" or "difference" question needs row-by-row subtraction — never average or otherwise combine the two series directly. If a table gives a grand total and leaves one entry blank, find the missing value by subtracting the sum of the known entries from the given total. After answering, sanity-check the result against the direction the data is actually moving — if every year in a table shows rising sales, a computed "decrease" should make you go back and recheck a step.

Examples

- A company’s annual sales (in ₹ lakhs) were: 2020 – 50, 2021 – 60, 2022 – 75, 2023 – 90, 2024 – 120. Find the percentage increase in sales from 2020 to 2021. Solution: Increase = 60 – 50 = 10. Percentage increase = (10/50) × 100 = 20%. Answer: 20%
- Using the same sales table (2020 – 50, 2021 – 60, 2022 – 75, 2023 – 90, 2024 – 120, in ₹ lakhs), find the average annual sales over the five years. Solution: Sum = 50 + 60 + 75 + 90 + 120 = 395. Average = 395 / 5 = 79. Answer: ₹79 lakh
- Using the same sales table (2020 – 50, 2021 – 60, 2022 – 75, 2023 – 90, 2024 – 120, in ₹ lakhs), in which year was the percentage increase in sales over the previous year the highest? Solution: 2020→2021: (10/50)×100 = 20%. 2021→2022: (15/60)×100 = 25%. 2022→2023: (15/75)×100 = 20%. 2023→2024: (30/90)×100 ≈ 33.3%. The highest is 2023→2024, so 2024 had the highest percentage increase over its previous year. Answer: 2024
- Using the same sales table (2020 – 50, 2021 – 60, 2022 – 75, 2023 – 90, 2024 – 120, in ₹ lakhs), find the ratio of sales in 2022 to sales in 2024. Solution: Ratio = 75 : 120. Dividing both terms by their HCF, 15: 75÷15 : 120÷15 = 5 : 8. Answer: 5 : 8
- A company's sales and costs (₹ lakh) were: 2021 – Sales 60, Cost 45; 2022 – Sales 75, Cost 50. Find the profit margin (profit as a percentage of sales) for 2022. Solution: Profit for 2022 = Sales – Cost = 75 – 50 = 25. Profit margin = (Profit / Sales) × 100 = (25 / 75) × 100 ≈ 33.3% Answer: ≈ 33.3%
- A shop's monthly sales (₹ thousand) for January to May totalled ₹310. The figures for January, February, April and May were 40, 55, 65 and 70 respectively, and March's figure is missing. Find March's sales. Solution: Sum of the known four months = 40 + 55 + 65 + 70 = 230. March's sales = total – known sum = 310 – 230 = ₹80 thousand Answer: ₹80 thousand
- A store sells the same product across 3 regions: Region A sold 200 units at ₹50 each, Region B sold 150 units at ₹60 each, and Region C sold 100 units at ₹80 each. Find the average selling price per unit across all three regions combined. Solution: This needs a weighted average (by units sold), not a simple average of the three prices. Total revenue = (200×50) + (150×60) + (100×80) = 10,000 + 9,000 + 8,000 = 27,000. Total units = 200 + 150 + 100 = 450. Average price per unit = 27,000 / 450 = ₹60. Answer: ₹60 per unit
- Practice: Using the same sales table (2020: 50, 2021: 60, 2022: 75, 2023: 90, 2024: 120, in ₹ lakhs), find the percentage increase in sales from 2022 to 2023. (Answer: 20%)
- Practice: Using the same sales table, find the total sales over 2020, 2021 and 2022 combined. (Answer: ₹185 lakh)
- Practice: Using the same sales table, find the ratio of sales in 2020 to sales in 2023. (Answer: 5 : 9)
- Practice: A company's sales and cost for 2023 were ₹90 lakh and ₹65 lakh respectively. Find the profit margin (profit as a percentage of sales). (Answer: ≈ 27.8%. Profit = 25; (25/90) × 100.)
- Practice: A 4-quarter table totals ₹500 crore. Q1, Q2 and Q3 were ₹100, ₹140 and ₹130 crore. Find Q4. (Answer: ₹130 crore. 500 – (100+140+130).)
- Practice: A store sells 3 products: Product X — 50 units at ₹20 each; Product Y — 30 units at ₹40 each; Product Z — 20 units at ₹100 each. Find the average price per unit sold across all three. (Answer: ₹42. Total revenue = 1,000+1,200+2,000 = 4,200; total units = 100.)
- Practice: Using the original sales table (2020–2024: 50, 60, 75, 90, 120, totalling 395), find what percentage of the 5-year total came from 2024 alone. (Answer: ≈ 30.4%. (120/395) × 100.)

Quick revision: Always check the units and any scale given with the table or chart ("figures in thousands," "sales in ₹ lakhs") before calculating — misreading the scale is the single most common careless mistake in this topic; For "what percentage more/less" questions, calculate the percentage change relative to the ORIGINAL value (the thing being compared against), not the new value — the same base-of-comparison rule from Percentages, just inside a table; For multi-step questions, compute each individual value needed FIRST and write it down, rather than trying to combine several lookups in your head at once; A ratio or an average computed from table data works exactly like the standalone Ratio/Average topics — the table just supplies the raw numbers; When a table mixes two related series for the same categories (e.g. sales AND cost, or income AND expenditure), a "profit" or "difference" question needs row-by-row subtraction — never average or otherwise combine the two series directly; If a table gives a grand total and leaves one entry blank, find the missing value by subtracting the sum of the known entries from the given total; After answering, sanity-check the result against the direction the data is actually moving — if every year in a table shows rising sales, a computed "decrease" should make you go back and recheck a step

Pie Chart Solved Questions

A pie chart is Data Interpretation with the categories expressed as a share of 360° instead of a plain table — the arithmetic underneath is the same percentage work, just one extra conversion step when a question gives degrees instead of a percentage directly. This page covers that conversion, comparing categories by ratio, the risk of comparing percentages across two different charts, and finding a missing category, with several fully worked examples. Every pie chart totals 360° (a full circle) and 100% — each category's slice is a percentage of the whole, or equivalently a fraction of 360°. If a category's percentage is given directly, its value = (percentage/100) × total. If its angle (in degrees) is given instead, first convert: percentage = (angle/360) × 100. To combine two or more categories, add their percentages (or their degrees) first, then apply the total once — don't calculate each category's value separately and add those, since rounding can creep in. To find a category that isn't stated directly, subtract the sum of the given percentages (or degrees) from 100% (or 360°). To compare two categories within the SAME pie chart, you can skip converting to rupee values entirely and just compare their angles or percentages as a ratio — e.g. if Rent is 30% and Food is 25% of the same chart, Rent : Food = 30 : 25 = 6 : 5, regardless of what the total actually is. Comparing the SAME category across two DIFFERENT pie charts (such as two different years) by percentage alone can be misleading if the totals differ — a smaller percentage of a much larger total can still be a bigger actual amount than a larger percentage of a smaller total. Always convert both to actual values before comparing across charts.

Examples

- A family's monthly expenses of ₹40,000 are distributed as: Rent 30%, Food 25%, Transport 15%, Savings 20%, Other 10%. Find the amount spent on Food. Solution: Food = 25% of ₹40,000. = (25/100) × 40,000 = ₹10,000. Answer: ₹10,000
- Using the same expense distribution (Rent 30%, Food 25%, Transport 15%, Savings 20%, Other 10%, total ₹40,000), find the amount spent on Rent and Transport combined. Solution: Combine the percentages first: Rent + Transport = 30% + 15% = 45%. 45% of ₹40,000 = (45/100) × 40,000 = ₹18,000. Answer: ₹18,000
- In a pie chart, the angle representing a company's Marketing budget is 72°. If the total budget is ₹15,00,000, find the amount allocated to Marketing. Solution: Convert the angle to a percentage: (72/360) × 100 = 20%. 20% of ₹15,00,000 = (20/100) × 15,00,000 = ₹3,00,000. Answer: ₹3,00,000
- Using the same company budget (total ₹15,00,000), if R&D is allocated 90° and Operations is allocated 108°, find the combined percentage of R&D and Operations. Solution: Combine the degrees first: 90° + 108° = 198°. Convert to a percentage: (198/360) × 100 = 55%. Answer: 55%
- Using the same family expense chart (Rent 30%, Food 25%, Transport 15%, Savings 20%, Other 10%), find the ratio of the amount spent on Rent to the amount spent on Food. Solution: Since both are percentages of the SAME total, their ratio is just the ratio of the percentages: 30 : 25. Simplify: 30 : 25 = 6 : 5. Answer: 6 : 5
- In Year 1 (total budget ₹10,00,000), Marketing was 20% of the budget. In Year 2 (total budget ₹15,00,000), Marketing was 15% of the budget. Did the actual amount spent on Marketing increase or decrease? Solution: Year 1 amount = 20% of 10,00,000 = ₹2,00,000. Year 2 amount = 15% of 15,00,000 = ₹2,25,000. Even though the percentage fell from 20% to 15%, the actual amount spent increased (₹2,00,000 → ₹2,25,000), because the total budget grew by more than the percentage shrank. Answer: Increased (₹2,00,000 to ₹2,25,000), despite the percentage falling
- In a pie chart, four categories take up 80°, 100°, 70° and 60°. Find the degree and the percentage of the fifth (unstated) category. Solution: Sum of the known degrees = 80 + 100 + 70 + 60 = 310°. Remaining degree = 360° − 310° = 50°. Percentage = (50/360) × 100 ≈ 13.9%. Answer: 50°, ≈ 13.9%
- Practice: Using the same family expense distribution (Rent 30%, Food 25%, Transport 15%, Savings 20%, Other 10%, total ₹40,000), find the amount spent on Savings. (Answer: ₹8,000)
- Practice: In a pie chart, the angle representing a department's budget is 54°. If the total budget is ₹20,00,000, find the amount allocated. (Answer: ₹3,00,000)
- Practice: Using the same company budget (total ₹15,00,000) with Marketing at 72°, R&D at 90°, and Operations at 108°, find the degree allocated to the remaining categories combined. (Answer: 90°)
- Practice: Using the same family expense chart, find the ratio of the amount spent on Transport to the amount spent on Savings. (Answer: 3 : 4. Ratio of percentages: 15:20.)

- Practice: In Year 1 (total ₹8,00,000), R&D was 25% of the budget. In Year 2 (total ₹6,00,000), R&D was 30%. Find the actual amount in each year and state whether it increased or decreased. (Answer: Year 1 = ₹2,00,000; Year 2 = ₹1,80,000 — decreased, despite the percentage rising from 25% to 30%.)
- Practice: In a pie chart, four categories take up 90°, 60°, 75° and 45°. Find the degree of the fifth category. (Answer: 90°. 360 − (90+60+75+45).)
- Practice: In a pie chart representing a budget of ₹24,00,000, a department is allocated 45°. Find the amount allocated. (Answer: ₹3,00,000. (45/360)×100 = 12.5%; 12.5% of 24,00,000.)

> **Quick revision:** Every pie chart totals 360° (a full circle) and 100% — each category's slice is a percentage of the whole, or equivalently a fraction of 360°; If a category's percentage is given directly, its value = (percentage/100) × total. If its angle (in degrees) is given instead, first convert: percentage = (angle/360) × 100; To combine two or more categories, add their percentages (or their degrees) first, then apply the total once — don't calculate each category's value separately and add those, since rounding can creep in; To find a category that isn't stated directly, subtract the sum of the given percentages (or degrees) from 100% (or 360°); To compare two categories within the SAME pie chart, you can skip converting to rupee values entirely and just compare their angles or percentages as a ratio — e.g. if Rent is 30% and Food is 25% of the same chart, Rent : Food = 30 : 25 = 6 : 5, regardless of what the total actually is; Comparing the SAME category across two DIFFERENT pie charts (such as two different years) by percentage alone can be misleading if the totals differ — a smaller percentage of a much larger total can still be a bigger actual amount than a larger percentage of a smaller total. Always convert both to actual values before comparing across charts

Bar Graph Solved Questions

A bar graph is Data Interpretation with each category's value read directly off a bar's height instead of a table cell — once you've read the numbers correctly, it's the same percentage, average and ratio work as every other DI format. This page covers that core idea, grouped (multi-series) bar graphs, and the misleading-axis caution, with several fully worked examples using shared data sets. Each bar's height (or length, for a horizontal bar graph) represents that category's value on the scale shown on the axis — read it carefully against the axis markings before calculating anything. A percentage change between two bars uses the same rule as any other percentage question: (new − old) / old × 100, with the EARLIER bar as the original value. For "which period had the highest/lowest change" questions, calculate every period-to-period change first and write them all down, rather than trying to eyeball which bars look most different. An average across several bars is just their sum divided by the count — the bar graph format doesn't change the formula, only how you read the input values. Some bar graphs show TWO or more bars per category (a "grouped" bar graph) — e.g. separate bars for Boys and Girls in each year. Keep the series separate throughout a calculation; only add or average across series if the question explicitly asks for a combined figure. Watch for a y-axis that doesn't start at zero (a "broken" or truncated axis) — bars can look dramatically different in height even when the underlying percentage difference between them is small. Always calculate from the actual labelled values on the axis, never from how tall the bars visually appear.

Examples

- A bar graph shows the number of books sold by a store over five months: January 120, February 150, March 90, April 180, May 160. Find the percentage increase in books sold from January to February. Solution: Increase = 150 − 120 = 30. Percentage increase = (30/120) × 100 = 25%. Answer: 25%
- Using the same data (January 120, February 150, March 90, April 180, May 160), find the average number of books sold per month over the five months. Solution: Sum = 120 + 150 + 90 + 180 + 160 = 700. Average = 700 / 5 = 140. Answer: 140 books
- Using the same data, find the ratio of books sold in March to books sold in April. Solution: Ratio = 90 : 180. Dividing both terms by their HCF, 90: 1 : 2. Answer: 1 : 2
- Using the same data, in which month-to-month transition was the percentage DECREASE the highest, and what was that percentage? Solution: Check every transition: Jan→Feb is an increase. Feb→Mar: decrease of 60, (60/150)×100 = 40%. Mar→Apr is an increase. Apr→May: decrease of 20, (20/180)×100 ≈ 11.1%. The highest decrease is Feb→Mar at 40%. Answer: February to March, a 40% decrease
- A grouped bar graph shows runs scored by two batsmen over 3 matches: Match 1: A = 40, B = 55; Match 2: A = 65, B = 50; Match 3: A = 50, B = 70. Find the total runs scored by each batsman. Solution: A's total = 40 + 65 + 50 = 155. B's

total = 55 + 50 + 70 = 175. Answer: A = 155 runs, B = 175 runs

- Using the same data, in which match was the difference between the two batsmen's scores the largest, and what was that difference? Solution: Match 1: |55−40| = 15. Match 2: |65−50| = 15. Match 3: |70−50| = 20. The largest difference is in Match 3, at 20 runs. Answer: Match 3, a difference of 20 runs
- A bar graph's y-axis starts at 80 (not 0). Two bars correspond to actual values of 92 and 96. A quick glance makes the second bar look like a huge jump because of the truncated axis. Find the actual percentage increase from the first bar to the second. Solution: Always compute from the real values, not the visual bar heights: increase = 96 − 92 = 4. Percentage increase = (4/92) × 100 ≈ 4.35%. This is a modest increase, even though a truncated axis starting at 80 can make it look dramatic. Answer: ≈ 4.35%
- Practice: Using the same data (January 120, February 150, March 90, April 180, May 160 books), find the percentage increase in books sold from March to April. (Answer: 100%)
- Practice: Using the same data, find the total books sold in January, February and March combined. (Answer: 360 books)
- Practice: Using the same data, find the ratio of books sold in January to books sold in May. (Answer: 3 : 4)
- Practice: A grouped bar graph shows sales (₹ thousand) of two products over 3 quarters: Q1: X=30, Y=45; Q2: X=50, Y=40; Q3: X=45, Y=65. Find the total sales for each product. (Answer: X = 125, Y = 150 (₹ thousand).)
- Practice: Using the same data, in which quarter was the difference between X and Y sales the largest? (Answer: Q3, a difference of 20 (65−45).)
- Practice: A bar graph's y-axis starts at 50 (not 0). Two bars correspond to actual values of 61 and 64. Find the actual percentage increase from the first to the second. (Answer: ≈ 4.92%. (64−61)/61 × 100.)
- Practice: Using the product sales data (Q1: X=30,Y=45; Q2: X=50,Y=40; Q3: X=45,Y=65), find the average quarterly sales of Product Y. (Answer: ₹50 thousand. (45+40+65)/3.)

> **Quick revision:** Each bar’s height (or length, for a horizontal bar graph) represents that category’s value on the scale shown on the axis — read it carefully against the axis markings before calculating anything; A percentage change between two bars uses the same rule as any other percentage question: (new − old) / old × 100, with the EARLIER bar as the original value; For "which period had the highest/lowest change" questions, calculate every period-to-period change first and write them all down, rather than trying to eyeball which bars look most different; An average across several bars is just their sum divided by the count — the bar graph format doesn’t change the formula, only how you read the input values; Some bar graphs show TWO or more bars per category (a "grouped" bar graph) — e.g. separate bars for Boys and Girls in each year. Keep the series separate throughout a calculation; only add or average across series if the question explicitly asks for a combined figure; Watch for a y-axis that doesn’t start at zero (a "broken" or truncated axis) — bars can look dramatically different in height even when the underlying percentage difference between them is small. Always calculate from the actual labelled values on the axis, never from how tall the bars visually appear

Line Graph Solved Questions

A line graph plots values over time (or another ordered variable) as points joined by straight segments — each point’s height is read off the same way a bar’s height is, and once you’ve read the numbers correctly it’s the same percentage, average and ratio work as any other Data Interpretation format. This page covers that core idea, multi-line graphs comparing two series, and why you shouldn't extrapolate a trend beyond the plotted points, with several fully worked examples. Each point on the line represents a value at that year (or period) on the scale shown on the axis — read it against the axis markings, not by guessing from the slope. A percentage change between two points uses the same rule as any other percentage question: (new − old) / old × 100, with the EARLIER point as the original value. A steep rising segment means a big increase and a steep falling segment means a big decrease — but "steep" is only a rough guide; for "which period had the largest change" questions, calculate every segment’s change first rather than eyeballing the slope. An average across several points is just their sum divided by the count, exactly like a bar graph or table — the format doesn’t change the formula, only how you read the input values. Some line graphs plot TWO or more lines on the same axes (e.g. the sales of two different companies over the same years) — read each line separately against the shared scale. A "when did A overtake B" question just means finding the first plotted point where A's value exceeds B's. Never extrapolate a trend beyond the plotted points. A line graph only

tells you what happened at the years actually shown — projecting the slope forward into an unshown year assumes the pattern continues exactly, which the chart itself does not confirm.

Examples

- A line graph shows the number of candidates (in thousands) who registered for an exam over five years: 2020 – 200, 2021 – 250, 2022 – 300, 2023 – 270, 2024 – 360. Find the percentage increase in registrations from 2020 to 2021. Solution: Increase = 250 − 200 = 50. Percentage increase = (50/200) × 100 = 25%. Answer: 25%
- Using the same data (2020 – 200, 2021 – 250, 2022 – 300, 2023 – 270, 2024 – 360), find the average number of candidates (in thousands) registered per year over the five years. Solution: Sum = 200 + 250 + 300 + 270 + 360 = 1380. Average = 1380 / 5 = 276. Answer: 276 thousand
- Using the same data, find the ratio of registrations in 2022 to registrations in 2024. Solution: Ratio = 300 : 360. Dividing both terms by their HCF, 60: 5 : 6. Answer: 5 : 6
- Using the same data, which year-to-year segment had the largest percentage change in either direction, and what was it? Solution: Check every segment: 2020→2021: +50, (50/200)×100 = 25%. 2021→2022: +50, (50/250)×100 = 20%. 2022→2023: −30, (30/300)×100 = 10% decrease. 2023→2024: +90, (90/270)×100 ≈ 33.3%. The largest change in magnitude is 2023→2024, at approximately 33.3%. Answer: 2023 to 2024, about a 33.3% increase
- A line graph shows annual profit (₹ lakh) of two companies: 2020: P=40, Q=55; 2021: P=60, Q=50; 2022: P=70, Q=65. In which year did Company P's profit first exceed Company Q's? Solution: In 2020, P (40) < Q (55). In 2021, P (60) > Q (50) for the first time. So 2021 is the first year P overtakes Q (and it stays ahead in 2022 as well: 70 > 65). Answer: 2021
- Using the same data, find the total combined profit of both companies in 2022. Solution: Total = P + Q in 2022 = 70 + 65 = ₹135 lakh. Answer: ₹135 lakh
- A line graph shows a company's sales rising from 100 (2020) to 130 (2021) to 160 (2022). Someone claims that in 2023, sales will "obviously" be 190. Is this a safe conclusion to draw directly from the graph? Solution: The graph only shows data through 2022, where the year-on-year increase happened to be a constant 30 units each time (100→130→160). Claiming 2023 will be 190 assumes that this exact 30-unit increase continues — but the graph itself provides no data point confirming that, and real-world trends can change due to factors the chart doesn't capture. So this is an assumption based on an observed pattern, not something the graph actually shows. Answer: No — it assumes the trend continues unchanged, which the graph does not confirm
- Practice: Using the same data (2020 – 200, 2021 – 250, 2022 – 300, 2023 – 270, 2024 – 360 thousand), find the percentage change in registrations from 2022 to 2023. (Answer: 10% decrease)
- Practice: Using the same data, find the total registrations (in thousands) across 2020, 2021 and 2022 combined. (Answer: 750 thousand)
- Practice: Using the same data, find the ratio of registrations in 2021 to registrations in 2024. (Answer: 25 : 36)
- Practice: A line graph shows revenue (₹ lakh) of two stores: 2020: A=35, B=50; 2021: A=55, B=48; 2022: A=65, B=60. In which year did Store A's revenue first exceed Store B's? (Answer: 2021. A (55) > B (48) for the first time; A was behind in 2020.)
- Practice: Using the same data, find the total combined revenue of both stores in 2021. (Answer: ₹103 lakh. 55 + 48.)
- Practice: A line graph shows a town's population rising from 10,000 (2018) to 12,000 (2019) to 14,500 (2020). Is it safe to assume the 2021 population will be exactly 17,000, based only on this graph? (Answer: No. The year-on-year increases (2,000, then 2,500) aren't even constant, so extending the pattern to exactly 17,000 is an unsupported assumption, not something the graph confirms.)
- Practice: Using the town population data (2018: 10,000; 2019: 12,000; 2020: 14,500), find the average population over the three years. (Answer: ≈ 12,166.67. (10,000+12,000+14,500)/3.)

Quick revision: Each point on the line represents a value at that year (or period) on the scale shown on the axis — read it against the axis markings, not by guessing from the slope; A percentage change between two points uses the same rule as any other percentage question: (new − old) / old × 100, with the EARLIER point as the original value; A steep rising segment means a big increase and a steep falling segment means a big decrease — but "steep" is only a rough guide; for "which period had the largest change" questions, calculate every segment's change first rather than eyeballing the slope; An average across several points is just their sum divided by the count, exactly like a bar graph or table — the format doesn't change the formula, only how you read the input values; Some line graphs plot TWO or more lines on the same axes (e.g. the sales of two different companies over the same years) — read each line separately against the shared scale. A "when did A overtake B" question just means finding the first plotted point where A's value exceeds B's; Never extrapolate a trend beyond the plotted points. A line graph only tells you what happened at the years actually shown — projecting the slope forward into an unshown year assumes the pattern continues exactly, which the chart itself does not confirm

Free practice questions: parikshasaathi.com/q/data-interpretation/

Number System

Number System: Divisibility, Factors, Powers and Remainders

For a non-negative integer N and positive divisor d, write N = dq + r with an integer quotient q and $0 \le r < d$; r is the least amount to subtract to reach a multiple, while the least non-negative amount to add is 0 when r = 0 and d − r otherwise.

A number is divisible by 9 when its digit sum is divisible by 9, and by 11 when the difference between its alternate-digit sums is a multiple of 11, including zero.

A prime is an integer greater than 1 with exactly two positive factors, and if N has prime factorisation p^a × q^b × ..., then its number of positive factors is (a + 1)(b + 1)... and its factor sum is (1 + p + ... + p^a)(1 + q + ... + q^b)... .

A perfect square has only even prime exponents and a perfect cube has prime exponents divisible by 3; a fraction in lowest terms terminates in base 10 exactly when its denominator contains only primes 2 and 5, with the larger exponent determining its minimum decimal places.

For n! = 1 × 2 × ... × n, the exponent of a prime p is floor(n/p) + floor(n/p^2) + ... until the terms become zero, where floor means round down; trailing zeros are counted by the exponent of 5 because factors of 2 are more abundant.

Use a verified repeating units-digit cycle for powers, substitute a known remainder when evaluating integer expressions modulo a divisor, use n^2 for the sum of the first n positive odd integers and n(n + 1) for the first n positive even integers, and count consecutive integer indices from a through b as b − a + 1.

Examples

- Remainders and nearby multiples: 725 = 18 × 40 + 5, so the remainder is 5, the least subtraction is 5, and the least addition is 18 − 5 = 13; check that 725 − 5 = 720 and 725 + 13 = 738 are consecutive multiples of 18.
- Factors and perfect powers: 72 = $2^3 \times 3^2$ has (3 + 1)(2 + 1) = 12 positive factors; to make it a perfect square by multiplication, add one factor of 2, giving 72 × 2 = 144 = 12^2; to make it a perfect cube by multiplication, add one factor of 3, giving 72 × 3 = 216 = 6^3.
- Factorials and decimal places: 25! has floor(25/5) + floor(25/25) = 5 + 1 = 6 trailing zeros; separately, 7/40 has denominator $2^3 \times 5$, so three decimal places are required, and 7/40 = 175/1000 = 0.175.

Quick revision: N = dq + r; factor count = ∏(exponent + 1); square/cube exponents are multiples of 2/3; factorial prime exponent = Σ floor(n/p^j); odd sum = n^2; even sum = n(n + 1); inclusive index count = last − first + 1.

Number System Solved Questions

Number System questions reward knowing a handful of divisibility shortcuts and a few standard counting formulas — summation formulas, unit-digit cycles, trailing zeros in a factorial, and the number of factors of an integer — so you never need to compute a huge number in full. This page covers all of these, with several fully worked examples.

Divisibility rules let you check whether a number is divisible by a small number without doing the division: by 2 if the last digit is even; by 3 if the digit sum is divisible by 3; by 4 if the last two digits form a number divisible by 4; by 5 if the last digit is 0 or 5; by 9 if the digit sum is divisible by 9; by 11 if the difference between the sum of digits in odd and even positions (counting from the right) is divisible by 11. When a number N is divided by a divisor D, $N = D \times Q + R$, where Q is the quotient and R is the remainder, with $0 \le R < D$. This relationship is the foundation of every remainder question. Remainders behave predictably under addition and multiplication: you can take the remainder of each piece first, rather than computing a huge expression in full — useful for "find the remainder of a large power" questions, where the remainders of successive powers usually repeat in a short cycle. A number is a perfect square if every exponent in its prime factorization is even; it's a perfect cube if every exponent is a multiple of 3. Useful summation formulas: sum of the first n natural numbers = $n(n+1)/2$; sum of the first n squares = $n(n+1)(2n+1)/6$; sum of the first n cubes = $[n(n+1)/2]^2$. To find the unit digit of a^n, find the (short) cycle of unit digits that powers of a repeat through, then use n mod (cycle length) to pick the right position — if that remainder comes out to 0, it points to the LAST entry of the cycle, not necessarily a's own unit digit. Number of trailing zeros in n! = how many times 5 divides into the numbers from 1 to n (count multiples of 5, then 25, then 125, … and add). Count only the 5s, not the 2s — a factorial always has more factors of 2 than of 5, so the 5s are the limiting factor. Number of factors of N, once you have its prime factorization $N = p_1^a \times p_2^b \times \ldots$, is $(a+1) \times (b+1) \times \ldots$ — each exponent contributes its own count of choices (0 through that exponent), multiplied together.

Examples

- Check whether 4,572 is divisible by 9. Solution: Digit sum = 4 + 5 + 7 + 2 = 18. 18 is divisible by 9, so 4,572 is divisible by 9. Answer: Yes, divisible by 9
- Check whether 9,251 is divisible by 11. Solution: From the right: position 1 = 1, position 2 = 5, position 3 = 2, position 4 = 9. Sum of odd positions (1st, 3rd) = 1 + 2 = 3. Sum of even positions (2nd, 4th) = 5 + 9 = 14. Difference = $|3 - 14|$ = 11, which is divisible by 11 — so 9,251 is divisible by 11. Answer: Yes, divisible by 11
- Find the remainder when 7^{100} is divided by 5. Solution: Find the pattern of 7^n mod 5: 7^1 mod 5 = 2, 7^2 mod 5 = 4, 7^3 mod 5 = 3, 7^4 mod 5 = 1, and then it repeats — a cycle of length 4. 100 ÷ 4 leaves no remainder, so 7^{100} lands on the 4th position in the cycle: remainder 1. Answer: 1
- A number, when divided by 15, leaves a remainder of 7. Find the remainder when the same number is divided by 5. Solution: The number is $N = 15Q + 7$ for some quotient Q. Since 15 is a multiple of 5, the term 15Q is exactly divisible by 5. So the remainder on dividing N by 5 is just 7 mod 5 = 2. (Check with Q = 1: N = 22, and 22 ÷ 5 leaves remainder 2.) Answer: 2
- Find the sum of the first 20 natural numbers, the sum of their squares, and the sum of their cubes. Solution: Sum of first n numbers = $n(n+1)/2 = 20 \times 21 / 2 = 210$. Sum of first n squares = $n(n+1)(2n+1)/6 = (20 \times 21 \times 41) / 6 = 17{,}220 / 6 = 2{,}870$. Sum of first n cubes = $[n(n+1)/2]^2 = 210^2 = 44{,}100$. Answer: Sum = 210, Sum of squares = 2,870, Sum of cubes = 44,100
- Find the unit digit of 7^{95}. Solution: Unit digits of powers of 7 cycle every 4 steps: $7^1 \to 7$, $7^2 \to 9$, $7^3 \to 3$, $7^4 \to 1$, then repeat. $95 = 4 \times 23 + 3$, so the remainder is 3 — this points to the 3rd entry in the cycle. The 3rd entry is 3. Answer: 3
- Find the number of trailing zeros in 100!. Solution: Count multiples of 5 up to 100: $\lfloor 100/5 \rfloor = 20$. Count multiples of 25 up to 100 (each contributes one extra factor of 5): $\lfloor 100/25 \rfloor = 4$. Multiples of 125 up to 100: 0, since 125 > 100. Total trailing zeros = 20 + 4 = 24. Answer: 24
- Find the number of factors of 360. Solution: $360 = 2^3 \times 3^2 \times 5^1$. Number of factors = $(3+1) \times (2+1) \times (1+1) = 4 \times 3 \times 2 = 24$ Answer: 24
- Practice: Check whether 3,684 is divisible by 4. (Answer: Yes (last two digits, 84, are divisible by 4))
- Practice: Check whether 61,809 is divisible by 11. (Answer: Yes (odd-position sum 23 minus even-position sum 1 = 22, divisible by 11))
- Practice: A number, when divided by 24, leaves a remainder of 11. Find the remainder when the same number is divided by 8. (Answer: 3)
- Practice: Find the sum of the first 15 natural numbers and the sum of their squares. (Answer: Sum = 120, Sum of squares = 1,240. 15×16/2 = 120; (15×16×31)/6 = 7,440/6.)
- Practice: Find the unit digit of 8^{53}. (Answer: 8. Cycle of 8 is 8,4,2,6 (length 4); 53 = 4×13+1, so the 1st entry, which is 8.)

- Practice: Find the number of trailing zeros in 50!. (Answer: 12. $\lfloor 50/5 \rfloor + \lfloor 50/25 \rfloor = 10 + 2$.)
- Practice: Find the number of factors of 180. (Answer: 18. $180 = 2^2 \times 3^2 \times 5^1$; $(2+1)(2+1)(1+1) = 3 \times 3 \times 2$.)

Quick revision: Divisibility rules let you check whether a number is divisible by a small number without doing the division: by 2 if the last digit is even; by 3 if the digit sum is divisible by 3; by 4 if the last two digits form a number divisible by 4; by 5 if the last digit is 0 or 5; by 9 if the digit sum is divisible by 9; by 11 if the difference between the sum of digits in odd and even positions (counting from the right) is divisible by 11; When a number N is divided by a divisor D, $N = D \times Q + R$, where Q is the quotient and R is the remainder, with $0 \le R < D$. This relationship is the foundation of every remainder question; Remainders behave predictably under addition and multiplication: you can take the remainder of each piece first, rather than computing a huge expression in full — useful for "find the remainder of a large power" questions, where the remainders of successive powers usually repeat in a short cycle; A number is a perfect square if every exponent in its prime factorization is even; it's a perfect cube if every exponent is a multiple of 3; Useful summation formulas: sum of the first n natural numbers = $n(n+1)/2$; sum of the first n squares = $n(n+1)(2n+1)/6$; sum of the first n cubes = $[n(n+1)/2]^2$; To find the unit digit of a^n, find the (short) cycle of unit digits that powers of a repeat through, then use n mod (cycle length) to pick the right position — if that remainder comes out to 0, it points to the LAST entry of the cycle, not necessarily a's own unit digit; Number of trailing zeros in n! = how many times 5 divides into the numbers from 1 to n (count multiples of 5, then 25, then 125, … and add). Count only the 5s, not the 2s — a factorial always has more factors of 2 than of 5, so the 5s are the limiting factor; Number of factors of N, once you have its prime factorization $N = p_1\text{^}a \times p_2\text{^}b \times \ldots$, is $(a+1) \times (b+1) \times \ldots$ — each exponent contributes its own count of choices (0 through that exponent), multiplied together

Free practice questions: parikshasaathi.com/q/number-system/

Percentage

Percentage Conversions, Changes and Applications

Percent means per hundred: convert a fraction to a percentage by multiplying by 100, convert a percentage to a fraction by dividing by 100, and calculate a part as (percentage/100) × whole.

A percentage comparison or change must name its reference base: part/whole × 100 gives a share, while (new − old)/old × 100 gives a signed percentage change, positive for an increase and negative for a decrease.

An increase of p% multiplies the original by 1 + p/100 and a decrease by 1 − p/100; reverse the change by dividing the final value by the applicable multiplier. Successive changes multiply their factors, so two signed changes a% and b% combine to a + b + ab/100 percent, using negative values for decreases; equal increases and decreases therefore do not cancel.

For marks, divide total earned marks by total possible marks; for population, follow each explicitly stated percentage base and add or subtract any separately stated number of residents. Savings equal income minus expenditure, so changes in income and expenditure must be applied separately before comparing savings; if price changes but expenditure stays fixed, quantity changes in inverse proportion to price.

Examples

- Conversion and marks: 3/8 × 100 = 37.5%, and 37.5/100 reduces to 3/8; a score of 37.5% in a 640-mark exam is (3/8) × 640 = 240 marks, verified by 240/640 × 100 = 37.5%.
- Successive changes and reversal: an initial value of 800 rises by 20% to 960 and then falls by 10% of 960 to 864; the overall increase is (864 − 800)/800 × 100 = 8%, and the original is recovered by 864/(1.20 × 0.90) = 800.
- Income and expenditure: income of ₹40000 and expenditure of ₹30000 give savings of ₹10000; after a 20% income increase and a 10% expenditure increase, the amounts are ₹48000 and ₹33000, so savings become ₹15000, an increase of (15000 − 10000)/10000 × 100 = 50%.

Quick revision: p% = p/100; share = part/whole × 100; change% = (new − old)/old × 100; new = old × (1 ± p/100); original = new ÷ multiplier; successive multipliers multiply; savings = income − expenditure.

Percentage Solved Questions

Learn how to choose the correct percentage base, reverse a percentage change, combine successive changes and distinguish percentages from percentage points. Work from simple calculations to comparison and data-interpretation problems, then check your method with the practice questions. "Percent" just means "per hundred" — x% of a number N is (x/100) × N. Percentage change (increase or decrease) is always calculated on the original value: (change / original value) × 100. For two successive percentage changes of a% and b% (either can be negative, for a decrease), the net change is a + b + (ab/100) — not simply a + b. This is the single most common mistake in percentage questions. Before calculating, name the base: “A is 20% more than B” uses B as 100%. Reversing the comparison changes the base, so the reverse percentage is different. A percentage multiplier is 1 + r/100 for an increase and 1 − r/100 for a decrease. To recover an original value, divide the final value by the multiplier; do not subtract the same percentage from the final value. A change in a rate from 40% to 50% is 10 percentage points, but a 25% relative increase: (50 − 40)/40 × 100. State which measure you are using. For a combined percentage, add the underlying quantities before dividing. Averaging percentages is valid only when their bases are equal. Useful checks: an increase followed by an equal percentage decrease leaves a loss; a value after a discount must be below its original value; a percentage of a total cannot exceed 100% unless the comparison allows it.

Examples

- What is 35% of 240? Solution: 35% of 240 = (35/100) × 240 = 35 × 2.4 = 84 Answer: 84
- The price of an item increased from ₹800 to ₹920. What is the percentage increase? Solution: Increase = 920 − 800 = 120 Percentage increase = (120 / 800) × 100 = 15% Answer: 15%
- A number is first increased by 20%, then the result is decreased by 20%. What is the net percentage change? Solution: Using the successive-change formula: net change = a + b + (ab/100) = 20 + (−20) + (20 × −20)/100 = 0 − 4 = −4% Check directly: start with 100 → increase 20% → 120 → decrease 20% → 120 × 0.8 = 96, which is 4% less than 100. Answer: Net decrease of 4% (not 0%, even though +20 and −20 look like they should cancel)
- In an election between two candidates, the winner received 60% of the votes and won by 4,800 votes. Find the total number of votes polled. Solution: Winner: 60%, Loser: 40% → difference = 20% of total votes 20% of total = 4,800 1% of total = 240 100% of total = 24,000 Answer: 24,000 votes
- A salary becomes ₹28,800 after a 20% increase. What was the original salary? Solution: Final = original × 1.20. Original = 28,800 ÷ 1.20 = 24,000. Check: 20% of 24,000 is 4,800; adding it gives 28,800. Subtracting 20% of 28,800 would use the wrong base. Answer: ₹24,000
- A is 25% greater than B. By what percentage is B less than A? Solution: Let B = 100, so A = 125. The difference is 25, but the requested comparison uses A as the base. 25/125 × 100 = 20%. Answer: 20%
- The price of rice rises by 25%. By what percentage should consumption fall to keep spending unchanged? Solution: Spending = price × quantity. Let the old price and quantity each be 100. New price = 125. To keep the product 10,000, new quantity = 10,000/125 = 80. Quantity falls from 100 to 80, a 20% reduction. Answer: 20%
- One class has 40 students with a 75% pass rate; another has 60 students with a 50% pass rate. Find the combined pass rate. Solution: Passed in first class = 40 × 0.75 = 30. Passed in second class = 60 × 0.50 = 30. Combined = 60 passes out of 100 students = 60%. The unweighted average 62.5% is wrong because the class sizes differ. Answer: 60%
- A pass rate rises from 60% to 72%. Find the increase in percentage points and the relative percentage increase. Solution: Percentage-point change = 72 − 60 = 12. Relative change uses the old rate: 12/60 × 100 = 20%. Both describe the same change using different units. Answer: 12 percentage points; 20% relative increase
- Practice: What is 45% of 160? (Answer: 72. Split 45% into 40% + 5%: 64 + 8 = 72.)
- Practice: A number increased from 250 to 300. Find the percentage increase. (Answer: 20%. The increase is 50 and the original base is 250: 50/250 × 100.)
- Practice: If 20% of a number is 50, find the number. (Answer: 250. Divide 50 by 0.20.)
- Practice: A price after a 15% discount is ₹1,020. Find the original price. (Answer: ₹1,200. The final price is 85% of the original: 1,020/0.85 = 1,200.)

- Practice: A value rises by 10% and then 20%. Find the net increase. (Answer: 32%. Multiply 1.10 × 1.20 = 1.32; adding 10 and 20 ignores the changed base.)
- Practice: A value falls by 20%. What increase restores it? (Answer: 25%. Start with 100 → 80. The required gain is 20 on a base of 80: 20/80 × 100.)
- Practice: A rate changes from 8% to 10%. State both measures of increase. (Answer: 2 percentage points; 25% relative increase because 2/8 × 100 = 25.)
- Practice: 30% of 200 candidates and 50% of 100 candidates pass. What percentage passes overall? (Answer: 36⅔%. Passes = 60 + 50 = 110; total candidates = 300; 110/300 × 100.)

> **Quick revision:** "Percent" just means "per hundred" — x% of a number N is (x/100) × N; Percentage change (increase or decrease) is always calculated on the original value: (change / original value) × 100; For two successive percentage changes of a% and b% (either can be negative, for a decrease), the net change is a + b + (ab/100) — not simply a + b. This is the single most common mistake in percentage questions; Before calculating, name the base: “A is 20% more than B” uses B as 100%. Reversing the comparison changes the base, so the reverse percentage is different; A percentage multiplier is 1 + r/100 for an increase and 1 − r/100 for a decrease. To recover an original value, divide the final value by the multiplier; do not subtract the same percentage from the final value; A change in a rate from 40% to 50% is 10 percentage points, but a 25% relative increase: (50 − 40)/40 × 100. State which measure you are using; For a combined percentage, add the underlying quantities before dividing. Averaging percentages is valid only when their bases are equal; Useful checks: an increase followed by an equal percentage decrease leaves a loss; a value after a discount must be below its original value; a percentage of a total cannot exceed 100% unless the comparison allows it

Free practice questions: parikshasaathi.com/q/percentage-applications/

Ratio and Proportion

Ratios, Proportions, Sharing and Ages

A ratio a:b compares quantities in a stated order; first convert measurements to the same unit, clear fractions or decimals by multiplying both terms equally, and divide integer terms by their greatest common divisor to obtain lowest terms. Equivalent ratios multiply or divide every term by the same nonzero factor, and a:b = c:d implies ad = bc. To divide a total T in the ratio a:b, assign shares Ta/(a + b) and Tb/(a + b), extending the same parts method to three or more quantities; if a difference is given instead, match it to the difference in ratio parts. The compound ratio of a:b and c:d is ac:bd, linked ratios must give their shared quantity the same number of parts, and a positive mean proportional x between a and b satisfies $x^2 = ab$. In direct proportion y/x stays constant, whereas in inverse proportion xy stays constant; sharing inversely to a and b therefore uses the ratio 1/a:1/b = b:a. For age problems, write present ages as multiples of a common variable, add the same elapsed time to each age for a future ratio or subtract it for a past ratio, and check that all ages at the stated times are valid.

When a DIFFERENCE (not a total) is given alongside a ratio, match the known difference to the difference in ratio PARTS, not the full ratio terms: for a 3:5 ratio with an actual difference of 20, the ratio parts differ by 5–3 = 2, so each part is worth 20/2 = 10, giving actual values of 30 and 50.

A compound ratio simply multiplies corresponding terms of two separate ratios together: combining a:b and c:d gives ac:bd - useful when a final quantity depends on two independent ratio relationships simultaneously.

Examples

- Simplification and sharing: 1.2:1.8 = 12:18 = 2:3; dividing ₹1500 in that ratio gives 5 total parts worth ₹300 each, so the shares are ₹600 and ₹900.
- Linking and inverse proportion: if A:B = 2:3 and B:C = 4:5, make B equal to 12 in both ratios to obtain A:B:C = 8:12:15; separately, if 12 identical workers need 18 days for a fixed job, 27 equally productive workers need 216/27 = 8 days.
- Ages: if Asha:Bharat = 2:3 now and their ratio after 4 years is 5:7, write (2k + 4)/(3k + 4) = 5/7; this gives k = 8 and the present ages are 16 and 24.
- Find the mean proportional between 4 and 16. Worked solution: $x^2 = 4 \times 16 = 64$, so x = 8 (taking the positive root). Check: 4:8 = 1:2 and 8:16 = 1:2, the same ratio on both sides, confirming 8 is the mean proportional.

- Two numbers are in the ratio 3:5, and their difference is 20. Find the two numbers. Worked solution: The ratio parts differ by 5−3 = 2, and this corresponds to the actual difference 20, so each part is worth 20/2 = 10. The numbers are 3×10=30 and 5×10=50. Check: 50−30 = 20.
- Find the compound ratio of 2:3 and 4:5. Worked solution: Multiply corresponding terms: (2×4):(3×5) = 8:15.

> **Quick revision:** Ratio a:b compares in stated order; simplify to lowest terms. a:b=c:d implies ad=bc. Sharing a total T in ratio a:b gives shares Ta/(a+b) and Tb/(a+b). A difference matches the DIFFERENCE in ratio parts, not the full terms. Compound ratio of a:b and c:d = ac:bd. Mean proportional x between a,b: x^2=ab. Inverse sharing uses 1/a:1/b = b:a. Ages: express as kx, shift all ages by the same elapsed time for past/future ratios.

Ratio and Proportion Solved Questions

Ratio and Proportion is foundational — ages, mixtures, partnerships and even some Profit and Loss questions all lean on it. This page covers the core rules, direct versus inverse proportion, comparing and combining ratios, several fully worked examples, and practice questions with explained answers. A ratio a : b compares two quantities of the same kind and has no units. Multiplying or dividing both terms by the same non-zero number gives an equivalent ratio. The parts of a ratio are not the actual quantities unless a total or one actual value is also given — a : b = 3 : 5 really means the two quantities are 3x and 5x for some common multiplier x, which must be found from the rest of the question. A proportion is two equal ratios: a : b = c : d, which means a × d = b × c ("product of extremes = product of means") — this cross-multiplication is how most proportion questions get solved. To split a total T in the ratio a : b : c, each part = (that term ÷ sum of terms) × T. To combine two ratios a : b and b : c that share a term into one three-term ratio a : b : c, scale each ratio so the shared term (b) matches in both, usually by finding their LCM. Two quantities are in direct proportion if their ratio stays constant (a/b = k); they are in inverse proportion if their product stays constant (a × b = k). Mixing these up is a common mistake — for a fixed distance, speed and time are inversely related, not directly, because a higher speed means less time, not more. To compare two ratios a : b and c : d, cross-multiply: a × d versus b × c. If a × d > b × c, then a : b > c : d. In a continued proportion a : b = b : c, the middle term b is the mean proportional between a and c, and b = √(a × c).

Examples

- Divide ₹720 between A and B in the ratio 5 : 4. Solution: Sum of ratio terms = 5 + 4 = 9 A's share = (5/9) × 720 = 400 B's share = (4/9) × 720 = 320 Check: 400 + 320 = 720 Answer: A gets ₹400, B gets ₹320
- If a : b = 3 : 4 and b : c = 8 : 9, find a : b : c. Solution: The shared term is b. In the first ratio b = 4, in the second b = 8 — scale the first ratio so its b also becomes 8: multiply both terms of 3 : 4 by 2, giving 6 : 8. The second ratio already has b = 8, so it stays 8 : 9. Combined: a : b : c = 6 : 8 : 9. Answer: a : b : c = 6 : 8 : 9
- The present ages of A and B are in the ratio 3 : 5. After 6 years, the ratio of their ages becomes 2 : 3. Find A's present age. Solution: Let present ages be A = 3x and B = 5x. After 6 years: (3x + 6) / (5x + 6) = 2 / 3 Cross-multiply: 3(3x + 6) = 2(5x + 6) → 9x + 18 = 10x + 12 → x = 6 A's present age = 3x = 18. Check: after 6 years, A = 24, B = 36, and 24 : 36 = 2 : 3. Answer: 18 years
- If x : y = 2 : 3, find the value of (3x + 2y) : (4x − y). Solution: Let x = 2k and y = 3k for some k. 3x + 2y = 3(2k) + 2(3k) = 6k + 6k = 12k 4x − y = 4(2k) − 3k = 8k − 3k = 5k Ratio = 12k : 5k = 12 : 5 Answer: 12 : 5
- 15 workers can finish a job in 8 days. How many workers, working at the same rate, are needed to finish the same job in 5 days? Solution: For a fixed amount of work, the number of workers and the number of days are inversely proportional — more workers means fewer days needed. Workers × Days = constant = 15 × 8 = 120. For 5 days: Workers = 120 / 5 = 24. Answer: 24 workers
- Which ratio is greater: 3 : 4 or 5 : 7? Solution: Cross-multiply: compare 3 × 7 with 4 × 5. 3 × 7 = 21, and 4 × 5 = 20. Since 21 > 20, 3 : 4 is the greater ratio. Answer: 3 : 4 is greater
- Find the mean proportional between 4 and 25. Solution: The mean proportional b between a and c satisfies b = √(a × c). b = √(4 × 25) = √100 = 10 Answer: 10
- Practice: Divide ₹1,500 among A, B and C in the ratio 2 : 3 : 5. (Answer: A = ₹300, B = ₹450, C = ₹750. Sum of parts = 10; each part is (ratio term/10) × 1,500.)
- Practice: If a : b = 2 : 3 and b : c = 5 : 7, find a : b : c. (Answer: a : b : c = 10 : 15 : 21. Scale 2:3 by 5 and 5:7 by 3 so b = 15 in both.)

- Practice: Two numbers are in the ratio 4 : 5. If their sum is 108, find the two numbers. (Answer: 48 and 60. Each part = 108/9 = 12; 4×12 and 5×12.)
- Practice: 12 men can complete a task in 9 days, working at a constant rate per person. In how many days will 18 men complete it? (Answer: 6 days. Men × Days = 108 (constant); 108/18 = 6.)
- Practice: Which ratio is greater: 7 : 9 or 11 : 13? (Answer: 11 : 13 is greater. Cross-multiply: 7×13 = 91 and 9×11 = 99; 99 > 91.)
- Practice: Find the mean proportional between 9 and 16. (Answer: 12. $\sqrt{(9 \times 16)} = \sqrt{144}$.)
- Practice: Two numbers are in the ratio 7 : 11. The smaller number is 49. Find the larger number. (Answer: 77. 49/7 = 7 is the common multiplier x; the larger number = 11 × 7.)

> **Quick revision:** A ratio a : b compares two quantities of the same kind and has no units. Multiplying or dividing both terms by the same non-zero number gives an equivalent ratio; The parts of a ratio are not the actual quantities unless a total or one actual value is also given — a : b = 3 : 5 really means the two quantities are 3x and 5x for some common multiplier x, which must be found from the rest of the question; A proportion is two equal ratios: a : b = c : d, which means a × d = b × c ("product of extremes = product of means") — this cross-multiplication is how most proportion questions get solved; To split a total T in the ratio a : b : c, each part = (that term ÷ sum of terms) × T; To combine two ratios a : b and b : c that share a term into one three-term ratio a : b : c, scale each ratio so the shared term (b) matches in both, usually by finding their LCM; Two quantities are in direct proportion if their ratio stays constant (a/b = k); they are in inverse proportion if their product stays constant (a × b = k). Mixing these up is a common mistake — for a fixed distance, speed and time are inversely related, not directly, because a higher speed means less time, not more; To compare two ratios a : b and c : d, cross-multiply: a × d versus b × c. If a × d > b × c, then a : b > c : d; In a continued proportion a : b = b : c, the middle term b is the mean proportional between a and c, and b = $\sqrt{(a \times c)}$

Mean, Third & Fourth Proportional

These three question types all ask you to complete a proportion with a missing term — the only real skill is setting up the right proportion statement before cross-multiplying. This page covers all three, how they connect to geometric progressions, and reverse problems (given the proportional value, find one of the original numbers), with several fully worked examples. Fourth proportional to a, b, c: find x such that a : b = c : x. Cross-multiplying gives x = (b × c) / a. Third proportional to a, b: find x such that a : b = b : x — note b is repeated. Cross-multiplying gives x = b^2 / a. Mean proportional between a and b: find x such that a : x = x : b — x is repeated this time. Cross-multiplying gives x^2 = a × b, so x = $\sqrt{(ab)}$. The three are easy to mix up because they all involve three given-or-implied numbers — write out the proportion statement explicitly first, with the unknown in its correct position, before cross-multiplying. All three describe a geometric progression (a sequence with a constant ratio between consecutive terms): the third proportional makes a, b, x into a GP; the mean proportional makes a, x, b into a GP. Recognising this gives you a fast sanity check — divide consecutive terms and confirm the ratio is the same each time. Reverse problems give you the proportional value itself and ask for one of the original three numbers — rearrange the same cross-multiplied equation to isolate whichever variable is now the unknown, rather than solving for the usual one. Check your answer by substituting it back into the original proportion as an actual ratio (e.g. 3:5 should equal 12:20 after simplifying) to confirm the two sides genuinely match.

Examples

- Find the fourth proportional to 3, 5 and 12. Solution: Set up: 3 : 5 = 12 : x. Cross-multiply: x = (5 × 12) / 3 = 60 / 3 = 20. Answer: 20
- Find the third proportional to 4 and 8. Solution: Set up: 4 : 8 = 8 : x. Cross-multiply: x = 8^2 / 4 = 64 / 4 = 16. Answer: 16
- Find the mean proportional between 4 and 25. Solution: Set up: 4 : x = x : 25. Cross-multiply: x^2 = 4 × 25 = 100, so x = $\sqrt{100}$ = 10. Answer: 10
- Find the fourth proportional to 6, 9 and 18. Solution: Set up: 6 : 9 = 18 : x. Cross-multiply: x = (9 × 18) / 6 = 162 / 6 = 27. Answer: 27
- Find the third proportional to 6 and 12, and verify that 6, 12 and your answer form a geometric progression. Solution: x = 12^2 / 6 = 144 / 6 = 24. Check as a GP: 12/6 = 2, and 24/12 = 2 — the same common ratio both times, confirming 6, 12, 24 is a genuine GP. Answer: 24

- The fourth proportional to 4, x and 15 is 30. Find x. Solution: Set up: 4 : x = 15 : 30. Cross-multiply: 4 × 30 = 15 × x → 120 = 15x → x = 8. Answer: 8
- The mean proportional between 5 and some number N is 20. Find N. Solution: Set up: 5 : 20 = 20 : N. Cross-multiply: 5N = 400, so N = 80. Check: the mean proportional of 5 and 80 is √(5 × 80) = √400 = 20. Matches. Answer: 80
- Practice: Find the third proportional to 5 and 10. (Answer: 20)
- Practice: Find the mean proportional between 9 and 16. (Answer: 12)
- Practice: Find the fourth proportional to 2, 7 and 10. (Answer: 35)
- Practice: Find the third proportional to 3 and 9, and verify 3, 9 and your answer form a geometric progression. (Answer: 27. Check: 9/3 = 3 and 27/9 = 3, the same ratio.)
- Practice: The fourth proportional to 5, y and 12 is 36. Find y. (Answer: 15. 5:y = 12:36, so 5×36 = 12y.)
- Practice: The mean proportional between 8 and some number N is 16. Find N. (Answer: 32. 8:16 = 16:N, so N = 256/8.)
- Practice: Find the third proportional to 7 and 21. (Answer: 63. x = 21²/7 = 441/7.)

> **Quick revision:** Fourth proportional to a, b, c: find x such that a : b = c : x. Cross-multiplying gives x = (b × c) / a; Third proportional to a, b: find x such that a : b = b : x — note b is repeated. Cross-multiplying gives x = b² / a; Mean proportional between a and b: find x such that a : x = x : b — x is repeated this time. Cross-multiplying gives x² = a × b, so x = √(ab); The three are easy to mix up because they all involve three given-or-implied numbers — write out the proportion statement explicitly first, with the unknown in its correct position, before cross-multiplying; All three describe a geometric progression (a sequence with a constant ratio between consecutive terms): the third proportional makes a, b, x into a GP; the mean proportional makes a, x, b into a GP. Recognising this gives you a fast sanity check — divide consecutive terms and confirm the ratio is the same each time; Reverse problems give you the proportional value itself and ask for one of the original three numbers — rearrange the same cross-multiplied equation to isolate whichever variable is now the unknown, rather than solving for the usual one; Check your answer by substituting it back into the original proportion as an actual ratio (e.g. 3:5 should equal 12:20 after simplifying) to confirm the two sides genuinely match

Free practice questions: parikshasaathi.com/q/ratio-proportion-applications/

Simple and Compound Interest

Simple and Compound Interest — Concepts and Applications

Simple interest is Prt/100 when principal P, annual percentage rate r and time t in years use consistent units, and amount equals principal plus interest. With annual compounding at a constant rate, amount is P(1 + r/100)^t for a whole number of years. Half-yearly compounding at a stated nominal annual rate uses half that rate per half-year and twice as many periods. Changing compound rates multiply their individual period factors, while simple interest at changing rates is always calculated on the unchanged principal. Percentage growth uses positive multipliers and depreciation uses retained fractions, so the same compounding method applies when those rules are explicitly stated.

For exactly 2 years, there's a direct shortcut for the difference between compound and simple interest: CI − SI = P × (r/100)² - this comes from the fact that compound interest effectively earns "interest on the first year's interest" in year two, which simple interest never does. This shortcut only holds for 2 years; longer periods need the full compound formula.

To find an unknown rate or time from a known simple interest, rearrange the same formula: rate = (SI × 100)/(P × t), or time = (SI × 100)/(P × r) - whichever quantity is missing, isolate it the same way you would in any linear equation.

Examples

- Find the simple interest in rupees on ₹5500 at 5% per year for 3 years. Worked solution: Simple interest = principal × annual rate × time/100. Thus SI = 5500 × 5 × 3/100 = ₹825.
- ₹11000 is invested at 10% per year compounded annually. What is the amount in rupees after 2 years? Worked solution: After year 1 the balance is ₹12100. Multiplying again by 1.10 gives ₹13310.

- A machine valued at ₹26000 loses exactly 10% of its current value at each year-end. Find its value in rupees after 2 years. Worked solution: Each year retains 90% of the previous value. Therefore value = 26000 × 0.90^2 = ₹21060.
- Find the difference between compound interest and simple interest on ₹8000 at 5% per annum for 2 years. Worked solution: Using the shortcut, CI−SI = P×$(r/100)^2$ = 8000×$(0.05)^2$ = 8000×0.0025 = ₹20. Check directly: SI = 8000×5×2/100 = ₹800; CI = 8000×1.05^2−8000 = ₹820; difference = ₹20, matching the shortcut.
- A sum of ₹1500 earns ₹450 simple interest in 3 years. Find the rate of interest per annum. Worked solution: Rate = (SI×100)/(P×t) = (450×100)/(1500×3) = 45000/4500 = 10%.
- ₹10000 is invested for 2 years at compound interest, with the first year's rate at 10% and the second year's rate at 20%. Find the final amount. Worked solution: Since the rates differ by year, multiply the individual year factors: amount = 10000 × 1.10 × 1.20 = ₹13200 - this is NOT the same as using a single averaged rate of 15% for both years.

> **Quick revision:** SI = Prt/100; CI amount = P(1 + r/100)^t for annual periods; half-yearly uses r/2 and 2t; changing rates multiply their period factors (never average them). For exactly 2 years: CI−SI = P×$(r/100)^2$. To find rate or time from SI: rearrange rate=(SI×100)/(Pt) or time=(SI×100)/(Pr).

Simple & Compound Interest

Interest questions are about keeping one distinction straight: simple interest is the same amount every year, compound interest grows on top of itself. This page covers both formulas, the fast difference shortcuts, what happens when the rate changes year to year, and several fully worked examples. Simple Interest (SI) = (P × R × T) / 100, where P is principal, R is the annual rate, T is time in years. Amount = P + SI. Compound Interest (CI), compounded annually: Amount = P × $(1 + R/100)^T$. CI = Amount − P. For the same P, R and T (with T > 1), CI is always greater than SI — interest earns interest in later years. When T = 1 year, CI and SI are identical, because no compounding has happened yet. Shortcut for 2 years: CI − SI = P × $(R/100)^2$. Shortcut for 3 years: CI − SI = P × $(R/100)^2$ × (3 + R/100). Both follow directly from expanding $(1 + R/100)^T$, and both are only valid for the stated number of years. Compounding over 2 years is the same idea as applying the same percentage increase twice in a row (see the Percentages lesson): the net percentage growth over 2 years is 2R + R^2/100, which is why CI for 2 years is always more than 2 × (one year's SI). If the rate changes each year (e.g. 10% in year 1, then 20% in year 2), do not use a single formula — multiply the year-by-year multipliers: Amount = P × $(1 + R_1/100)$ × $(1 + R_2/100)$ × ... A common mistake is plugging CI numbers into the SI formula (or vice versa) because a question mentions "interest" without saying which kind — always check the words "simple" or "compound" before choosing a formula. For a time period with a fraction of a year (e.g. 2 years 6 months) compounded annually, the standard convention used in these exams is: compute CI normally for the whole years, then add simple interest for the fractional part on the amount reached after the whole years — not compound interest on the fraction itself, unless a question says otherwise.

Examples

- Find the simple interest on ₹5,000 at 8% per annum for 3 years. Solution: SI = (P × R × T) / 100 = (5,000 × 8 × 3) / 100 = 1,200 Answer: ₹1,200
- A sum of ₹8,000 amounts to ₹9,440 in 3 years at simple interest. Find the rate of interest per annum. Solution: SI = 9,440 − 8,000 = 1,440 R = (SI × 100) / (P × T) = (1,440 × 100) / (8,000 × 3) = 144,000 / 24,000 = 6% Answer: 6% per annum
- Find the compound interest on ₹10,000 for 2 years at 10% per annum, compounded annually. Solution: Amount = 10,000 × $(1.10)^2$ = 10,000 × 1.21 = 12,100 CI = 12,100 − 10,000 = 2,100 Answer: ₹2,100
- Find the difference between the compound interest and simple interest on ₹15,000 for 2 years at 10% per annum. Solution: SI = (15,000 × 10 × 2) / 100 = 3,000 CI: Amount = 15,000 × $(1.10)^2$ = 15,000 × 1.21 = 18,150, so CI = 3,150 Difference = 3,150 − 3,000 = 150 Shortcut check: P × $(R/100)^2$ = 15,000 × 0.01 = 150 — matches, for 2 years only. Answer: ₹150
- Find the difference between the compound interest and simple interest on ₹8,000 for 3 years at 5% per annum. Solution: SI = (8,000 × 5 × 3) / 100 = 1,200 CI: Amount = 8,000 × $(1.05)^3$ = 8,000 × 1.157625 = 9,261, so CI = 1,261 Difference = 1,261 − 1,200 = 61 Shortcut check: P × $(R/100)^2$ × (3 + R/100) = 8,000 × 0.0025 × 3.05 = 20 × 3.05 = 61 — matches. Answer: ₹61

- A sum invested at 20% per annum, compounded annually, earns a compound interest of ₹1,760 in 2 years. Find the sum. Solution: Over 2 years at 20%, the amount factor is $(1.20)^2 = 1.44$, so CI is 0.44 of the principal. 0.44 × P = 1,760 P = 1,760 / 0.44 = 4,000 Answer: ₹4,000
- ₹5,000 is invested for 2 years with a rate of 10% in the first year and 20% in the second year, compounded annually. Find the compound interest earned. Solution: Amount = 5,000 × 1.10 × 1.20 (multiply each year's factor; do not average the two rates) = 5,000 × 1.32 = 6,600 CI = 6,600 − 5,000 = 1,600 Answer: ₹1,600
- Find the compound interest on ₹12,000 at 10% per annum for 2 years and 6 months, compounded annually. Solution: Amount after the first 2 whole years = 12,000 × $(1.10)^2$ = 12,000 × 1.21 = 14,520. For the remaining 6 months (0.5 year), add simple interest on this amount: SI = 14,520 × 10 × 0.5 / 100 = 726. Final amount = 14,520 + 726 = 15,246. CI = 15,246 − 12,000 = 3,246. Answer: ₹3,246
- Practice: Find the simple interest on ₹12,000 at 5% per annum for 4 years. (Answer: ₹2,400. SI = (12,000 × 5 × 4)/100.)
- Practice: A sum amounts to ₹6,600 in 4 years at 5% simple interest per annum. Find the principal. (Answer: ₹5,500. Amount = P(1 + 4×5/100) = 1.20P = 6,600.)
- Practice: Find the compound interest on ₹20,000 for 2 years at 5% per annum, compounded annually. (Answer: ₹2,050. Amount = 20,000 × 1.05^2 = 22,050.)
- Practice: Find the difference between the compound interest and simple interest on ₹4,000 for 3 years at 10% per annum. (Answer: ₹124. Using $P(R/100)^2(3+R/100)$ = 4,000 × 0.01 × 3.1.)
- Practice: A sum of ₹1,600 earns a compound interest of ₹900 in 2 years, compounded annually. Find the rate of interest. (Answer: 25%. $(1 + R/100)^2 = 1 + 900/1,600 = 1.5625 = 1.25^2$, so R = 25.)
- Practice: ₹2,000 is invested for 2 years at 25% in the first year and 10% in the second, compounded annually. Find the compound interest. (Answer: ₹750. Amount = 2,000 × 1.25 × 1.10 = 2,750.)
- Practice: Find the compound interest on ₹10,000 at 10% per annum for 1 year 6 months, compounded annually (using the standard fractional-year convention). (Answer: ₹1,550. Amount after 1 year = 11,000; SI on 11,000 for 0.5 year at 10% = 550; total CI = 1,000 + 550.)

> **Quick revision:** Simple Interest (SI) = (P × R × T) / 100, where P is principal, R is the annual rate, T is time in years. Amount = P + SI; Compound Interest (CI), compounded annually: Amount = P × $(1 + R/100)^T$. CI = Amount − P; For the same P, R and T (with T > 1), CI is always greater than SI — interest earns interest in later years. When T = 1 year, CI and SI are identical, because no compounding has happened yet; Shortcut for 2 years: CI − SI = P × $(R/100)^2$. Shortcut for 3 years: CI − SI = P × $(R/100)^2$ × (3 + R/100). Both follow directly from expanding $(1 + R/100)^T$, and both are only valid for the stated number of years; Compounding over 2 years is the same idea as applying the same percentage increase twice in a row (see the Percentages lesson): the net percentage growth over 2 years is $2R + R^2/100$, which is why CI for 2 years is always more than 2 × (one year's SI); If the rate changes each year (e.g. 10% in year 1, then 20% in year 2), do not use a single formula — multiply the year-by-year multipliers: Amount = P × $(1 + R_1/100)$ × $(1 + R_2/100)$ × ; A common mistake is plugging CI numbers into the SI formula (or vice versa) because a question mentions "interest" without saying which kind — always check the words "simple" or "compound" before choosing a formula; For a time period with a fraction of a year (e.g. 2 years 6 months) compounded annually, the standard convention used in these exams is: compute CI normally for the whole years, then add simple interest for the fractional part on the amount reached after the whole years — not compound interest on the fraction itself, unless a question says otherwise

Half-Yearly & Quarterly Compound Interest

The compound interest formula doesn’t change when compounding happens more often than once a year — only the rate and the time do, and getting that adjustment right is the entire skill in this topic. This page covers that adjustment, why more frequent compounding always earns slightly more, and reverse problems (given the amount, find the principal or rate), with several fully worked examples. For half-yearly compounding: divide the annual rate by 2, and multiply the time (in years) by 2, before applying Amount = P × (1 + rate/100)^periods. For quarterly compounding: divide the annual rate by 4, and multiply the time (in years) by 4. The underlying formula never changes — only what counts as "the rate per period" and "the number of periods" does. Get those two numbers right first, then it’s the same calculation as annual compounding. Compound Interest = Amount − Principal, exactly as with annual compounding — this last step is easy to forget after focusing on the rate/time adjustment. For the same principal, annual rate and time, more frequent compounding always earns slightly MORE interest: quarterly

beats half-yearly, which beats annual — because interest starts earning interest on itself sooner when periods are shorter. Reverse problems (given the final amount, find the principal or the rate) work exactly like annual compounding, just with the already-adjusted per-period rate and number of periods: Principal = Amount / (1 + rate/100)^periods, or compare the amount factor to standard powers to recover the rate.

Examples

- Find the compound interest on ₹10,000 for 1 year at 10% per annum, compounded half-yearly. Solution: Half-yearly: rate per period = 10/2 = 5%, number of periods = 1 × 2 = 2. Amount = 10,000 × $(1.05)^2$ = 10,000 × 1.1025 = ₹11,025. Compound Interest = 11,025 − 10,000 = ₹1,025. Answer: ₹1,025
- Find the amount on ₹10,000 for 6 months at 8% per annum, compounded quarterly. Solution: Quarterly: rate per period = 8/4 = 2%. 6 months = 1/2 year, so number of periods = (1/2) × 4 = 2. Amount = 10,000 × $(1.02)^2$ = 10,000 × 1.0404 = ₹10,404. Answer: ₹10,404
- Find the compound interest on ₹20,000 for 1 year at 20% per annum, compounded half-yearly. Solution: Half-yearly: rate per period = 20/2 = 10%, number of periods = 1 × 2 = 2. Amount = 20,000 × $(1.10)^2$ = 20,000 × 1.21 = ₹24,200. Compound Interest = 24,200 − 20,000 = ₹4,200. Answer: ₹4,200
- Find the compound interest on ₹15,000 for 1.5 years at 20% per annum, compounded half-yearly. Solution: Half-yearly: rate per period = 20/2 = 10%. 1.5 years means number of periods = 1.5 × 2 = 3. Amount = 15,000 × $(1.10)^3$ = 15,000 × 1.331 = ₹19,965. Compound Interest = 19,965 − 15,000 = ₹4,965. Answer: ₹4,965
- ₹10,000 is invested for 1 year at 20% per annum. Compare the compound interest earned if it is compounded annually versus half-yearly. Solution: Annually: Amount = 10,000 × 1.20 = ₹12,000, so CI = ₹2,000. Half-yearly: 2 periods at 10% each. Amount = 10,000 × $(1.10)^2$ = 10,000 × 1.21 = ₹12,100, so CI = ₹2,100. Half-yearly compounding earns ₹100 more, even with the same 20% annual rate — because the first half-year's interest starts earning its own interest in the second half. Answer: Annual CI = ₹2,000; Half-yearly CI = ₹2,100 (₹100 more)
- A sum, invested at 10% per annum compounded half-yearly, amounts to ₹9,261 after 1.5 years. Find the sum. Solution: Half-yearly: rate per period = 5%, number of periods = 1.5 × 2 = 3. Amount factor = $(1.05)^3$ = 1.157625. Principal = Amount / factor = 9,261 / 1.157625 = ₹8,000. Answer: ₹8,000
- A sum of ₹20,000, compounded quarterly for 6 months, amounts to ₹20,808. Find the annual rate of interest. Solution: Number of periods = (1/2) × 4 = 2. Amount factor = 20,808 / 20,000 = 1.0404. 1.0404 = $(1.02)^2$, so the rate per period = 2%. Annual rate = 2% × 4 = 8%. Answer: 8% per annum
- Practice: Find the compound interest on ₹12,000 for 1 year at 10% per annum, compounded half-yearly. (Answer: ₹1,230)
- Practice: Find the amount on ₹20,000 for 6 months at 8% per annum, compounded quarterly. (Answer: ₹20,808)
- Practice: Find the compound interest on ₹8,000 for 1.5 years at 20% per annum, compounded half-yearly. (Answer: ₹2,648)
- Practice: ₹15,000 is invested for 1 year at 10% per annum. Compare the compound interest if compounded annually versus half-yearly. (Answer: Annual CI = ₹1,500; Half-yearly CI = ₹1,537.50 (₹37.50 more).)
- Practice: A sum, invested at 20% per annum compounded half-yearly, amounts to ₹24,200 after 1 year. Find the sum. (Answer: ₹20,000. Amount factor = $(1.10)^2$ = 1.21; 24,200/1.21.)
- Practice: A sum of ₹12,000, compounded half-yearly for 1 year, amounts to ₹13,230. Find the annual rate of interest. (Answer: 10% per annum. Factor = 1.1025 = $(1.05)^2$, so rate per period = 5%, annual = 10%.)
- Practice: Find the amount on ₹5,000 for 1 year at 40% per annum, compounded half-yearly. (Answer: ₹7,200. Rate per period = 20%, periods = 2; 5,000 × $(1.20)^2$ = 5,000 × 1.44.)

Quick revision: For half-yearly compounding: divide the annual rate by 2, and multiply the time (in years) by 2, before applying Amount = P × (1 + rate/100)^periods; For quarterly compounding: divide the annual rate by 4, and multiply the time (in years) by 4; The underlying formula never changes — only what counts as "the rate per period" and "the number of periods" does. Get those two numbers right first, then it's the same calculation as annual compounding; Compound Interest = Amount − Principal, exactly as with annual compounding — this last step is easy to forget after focusing on the rate/time adjustment; For the same principal, annual rate and time, more frequent compounding always earns slightly MORE interest: quarterly beats half-yearly, which beats annual — because interest starts earning interest on itself sooner when periods are shorter; Reverse problems (given the final amount, find the principal or the rate) work exactly like annual compounding, just with the already-adjusted per-period rate and number of periods: Principal = Amount / (1 + rate/100)^periods, or compare the amount factor to standard powers to recover the rate

Free practice questions: parikshasaathi.com/q/simple-compound-interest/

Time and Work

Time and Work — Concepts and Applications

Treat one complete job as one unit, so a worker who finishes alone in d days has rate 1/d job per day under constant working conditions. Add rates for simultaneous independent work and subtract an emptying pipe's rate from filling rates. Split a changing workforce or alternate-day schedule into separate intervals, adding the completed fractions before finding the remaining time. Efficiency ratios are rate ratios, so completion times for the same job are inversely proportional to efficiency. When wages are explicitly proportional to work done, share them using each worker's rate multiplied by actual working time.

For questions about the NUMBER OF WORKERS needed rather than time, use the men-days constant: for a fixed amount of work, (number of workers) × (number of days) stays the same - if M1 workers take D1 days, then M2 workers take D2 days where M1×D1 = M2×D2. This is really the same "total work = rate × time" idea, just counting workers instead of a single rate.

Wage-sharing problems reduce to comparing rates, not days worked: if two people work together for the same duration, their share of the total payment is in the exact ratio of their individual rates (1/d for each), since they're both working the same amount of time but contributing work in proportion to how fast each of them is.

Examples

- Asha completes a job alone in 11 days at a constant rate. What fraction of the job does she complete in 3 days? Worked solution: Her daily rate is 1/11 of the job. In 3 days she completes 3 × 1/11 = 3/11. Check: the remaining 8 days account for fraction 8/11, and the two fractions sum to 1.
- Asha alone needs 8 working days and Bharat 12. They work on alternate working days, Asha first, one person at a time. What is the elapsed number of working days to finish, counting a final partial day proportionally? Worked solution: Each two-day cycle completes 1/8 + 1/12 = 5/24 of the job. After 4 full cycles, 1/6 remains. Apply Asha's rate first to that remainder, then Bharat's only if needed; completion occurs at 9.5 working days.
- Two inlets fill a tank alone in 12 and 6 hours. An outlet empties it alone in 24 hours. With constant rates, all three open on an empty tank. Find the filling time in hours. Worked solution: Net rate = 1/12 + 1/6 − 1/24 = 5/24 tank/hour. The time for one tank is 4.8 hours. Check in denominator 24: rates are 2/24, 4/24, and −1/24, totalling 5/24.
- If 6 men can build a wall in 10 days, how many men are needed to finish it in just 4 days (same work rate per man)? Worked solution: Using M1×D1 = M2×D2: 6×10 = M2×4, so M2 = 60/4 = 15 men.
- A can finish a job in 10 days and B in 15 days. They work together for the entire job and are paid ₹3000 total. How should the money be split? Worked solution: Rates are 1/10 and 1/15; combined rate = 1/6. A's share of the combined rate is (1/10)/(1/6) = 3/5, so A gets 3/5 × 3000 = ₹1800 and B gets the remaining 2/5 × 3000 = ₹1200.
- A is twice as efficient as B. If B alone takes 18 days to finish a job, how long does A alone take? Worked solution: Efficiency and time are inversely related, so A (twice as efficient) takes half the time: 18/2 = 9 days.
- A, B, and C can individually finish a job in 20, 30, and 60 days. Working together, how many days will they take? Worked solution: Combined rate = 1/20 + 1/30 + 1/60 = 3/60 + 2/60 + 1/60 = 6/60 = 1/10, so together they finish in 10 days.

> **Quick revision:** Work = rate × time; combined rates add, emptying rates subtract; account for each work interval separately; wages follow completed work when stated. For worker-count questions: M1×D1 = M2×D2 (men-days constant). Wage shares split in the exact ratio of individual rates when working the same duration.

Time and Work Solved Questions

Time and Work questions all rest on one idea: convert everyone's time into a daily work-rate, combine the rates, then convert back to time. This page covers that core idea, the men-days-work equivalence, alternate-day working and partial-participation variants that exams favour, and several fully worked examples. If a person finishes a piece of work in n days, their work rate is 1/n of the work per day. Converting everyone to a rate like this is the key move in almost every question here. When people work together, ADD their individual daily rates to get the combined rate. Time to finish together = 1 ÷ (combined rate). Shortcut for two people, A in x days and B in y days, working together from the start: combined time = xy / (x + y). (This is just the rate-addition rule worked out algebraically — useful to recall directly.) "X% more efficient" means X% more work done per day, i.e., the rate is multiplied by (1 + X/100) — and since rate and time are inverses, the time is divided by the same factor, not multiplied. For the same total work, men and days trade off: $Men_1 \times Days_1 = Men_2 \times Days_2$ (add × Hours per day on both sides if daily working hours change too). For a different amount of work W, the general form is (M × D × H) / W = constant. When two people work on alternate days, work out how much gets done in one full cycle (e.g. one day each), see how many whole cycles fit before the work is close to finished, then handle the leftover day by day — do not assume the alternation divides evenly unless it actually does. When someone leaves partway through (or joins partway through), split the problem into stages: find the work done in the stage where everyone present was working together, subtract it from 1, then solve the remaining stage with whoever is left. Check: after working through a multi-stage problem, add up every stage's contribution — it must total exactly 1 (one full job). If it doesn't, a stage was computed incorrectly.

Examples

- A can complete a piece of work in 12 days. What fraction of the work does he complete in 4 days? Solution: A's rate = 1/12 of the work per day. Work done in 4 days = 4 × (1/12) = 4/12 = 1/3 Answer: 1/3 of the work
- A can complete a work in 10 days and B can complete the same work in 15 days. If they work together, in how many days will they finish it? Solution: A's rate = 1/10, B's rate = 1/15 per day. Combined rate = 1/10 + 1/15 = 3/30 + 2/30 = 5/30 = 1/6 of the work per day. Time to finish together = 1 ÷ (1/6) = 6 days. Answer: 6 days
- A can do a piece of work in 20 days. B is 25% more efficient than A. Find the number of days B alone takes to complete the same work. Solution: "25% more efficient" means B's work rate is 1.25 times A's rate. Since time is inversely proportional to rate, B's time = A's time ÷ 1.25 = 20 ÷ 1.25 = 16 days Answer: 16 days
- A and B together can complete a work in 12 days. A alone can complete it in 20 days. In how many days can B alone complete the work? Solution: Combined rate (A + B) = 1/12. A's rate = 1/20. B's rate = 1/12 − 1/20 = 5/60 − 3/60 = 2/60 = 1/30 of the work per day. B's time alone = 1 ÷ (1/30) = 30 days. Answer: 30 days
- 12 men can build a wall in 15 days. How many men, working at the same rate, are needed to build it in 10 days? Solution: For the same total work, Men × Days stays constant: 12 × 15 = 180. For 10 days: Men = 180 / 10 = 18 men Answer: 18 men
- A can finish a work in 10 days and B in 15 days. They work on alternate days, starting with A. In how many days will the work finish? Solution: A's rate = 1/10, B's rate = 1/15. In each 2-day cycle (one day A, one day B), work done = 1/10 + 1/15 = 3/30 + 2/30 = 5/30 = 1/6. After 6 cycles (12 days), work done = 6 × 1/6 = 1 — exactly the whole job, with no leftover day to analyse. So the work finishes in exactly 12 days. Answer: 12 days
- A and B together can complete a work in 12 days. They work together for 4 days, after which A leaves, and B alone completes the remaining work in 16 more days. In how many days could B alone have completed the entire work? Solution: Combined rate = 1/12. Work done in the first 4 days = 4 × 1/12 = 1/3. Remaining work = 1 − 1/3 = 2/3, and B alone finishes this in 16 days, so B's rate = (2/3) / 16 = 1/24. B alone would take 1 ÷ (1/24) = 24 days for the entire work. Answer: 24 days
- Practice: A can complete a work in 8 days. What fraction of the work does he complete in 3 days? (Answer: 3/8 of the work.)
- Practice: A can complete a work in 6 days and B can complete the same work in 12 days. Working together, how many days will they take? (Answer: 4 days. Combined rate = 1/6 + 1/12 = 1/4.)

- Practice: A and B together can complete a work in 15 days. B alone can complete it in 25 days. In how many days can A alone complete the work? (Answer: 37.5 days. A's rate = 1/15 − 1/25 = 2/75; time = 75/2.)
- Practice: 15 men can complete a job in 8 days. How many men, at the same rate, are needed to complete it in 6 days? (Answer: 20 men. Men × Days = 120 (constant); 120/6 = 20.)
- Practice: A can finish a work in 8 days and B in 12 days. They work on alternate days, starting with A. In how many days will the work be completed? (Answer: 9.5 days. 4 full cycles (8 days) complete 5/6 of the work; day 9 (A) adds 1/8, leaving 1/24; B finishes that 1/24 in half a day at B's rate of 1/12 per day.)
- Practice: A and B can together finish a work in 15 days. They work together for 5 days, then A leaves and B alone finishes the remaining work in 20 more days. Find how long B alone would take for the entire work. (Answer: 30 days. Work done together in 5 days = 1/3; remaining 2/3 takes B 20 days, so B's rate = 1/30.)
- Practice: A can do a work in 18 days and B in 24 days. They work together for 8 days. What fraction of the work remains? (Answer: 2/9. Combined rate = 1/18 + 1/24 = 7/72; work done in 8 days = 56/72 = 7/9; remaining = 2/9.)

Quick revision: If a person finishes a piece of work in n days, their work rate is 1/n of the work per day. Converting everyone to a rate like this is the key move in almost every question here; When people work together, ADD their individual daily rates to get the combined rate. Time to finish together = 1 ÷ (combined rate); Shortcut for two people, A in x days and B in y days, working together from the start: combined time = xy / (x + y). (This is just the rate-addition rule worked out algebraically — useful to recall directly.); "X% more efficient" means X% more work done per day, i.e., the rate is multiplied by (1 + X/100) — and since rate and time are inverses, the time is divided by the same factor, not multiplied; For the same total work, men and days trade off: $Men_1 \times Days_1 = Men_2 \times Days_2$ (add × Hours per day on both sides if daily working hours change too). For a different amount of work W, the general form is (M × D × H) / W = constant; When two people work on alternate days, work out how much gets done in one full cycle (e.g. one day each), see how many whole cycles fit before the work is close to finished, then handle the leftover day by day — do not assume the alternation divides evenly unless it actually does; When someone leaves partway through (or joins partway through), split the problem into stages: find the work done in the stage where everyone present was working together, subtract it from 1, then solve the remaining stage with whoever is left; Check: after working through a multi-stage problem, add up every stage's contribution — it must total exactly 1 (one full job). If it doesn't, a stage was computed incorrectly

Pipes and Cisterns Solved Questions

Pipes and Cisterns is Time and Work with one twist — an outlet pipe drains the tank instead of filling it, so its rate is negative. This page covers that core idea, reverse problems (given the combined time, find one pipe's own time), staggered starts, and a pipe being closed partway through, with several fully worked examples. A pipe that FILLS a tank has a positive rate (fraction of the tank filled per hour); a pipe that EMPTIES it (an outlet) has a negative rate. Combine rates exactly like Time and Work, but outlet pipes subtract. If a filling pipe can fill a tank in x hours, its rate is 1/x. If an outlet can empty a full tank in y hours, its rate is −1/y. With multiple pipes open together, add up all the rates (filling positive, emptying negative) to get the net rate. Time to fill (or empty) = 1 ÷ |net rate|, and the sign tells you which direction the tank is actually moving. A pipe that's "twice as fast" as another has half the time — rate and time are inverses here just like in Time and Work. Reverse problems (given the combined time, find one pipe's own time) just invert the usual setup: rearrange 1/x ± 1/y = 1/(combined time) algebraically for the unknown rate, then take its reciprocal to get that pipe's own time. If one pipe starts before another joins in, or a pipe is closed partway through, split the problem into stages exactly like in Time and Work: find the fraction of the tank filled in each stage, subtract running totals from 1, and solve the final stage with whichever pipe(s) remain.

Examples

- Pipe A can fill a tank in 6 hours, and Pipe B can fill the same tank in 8 hours. If both pipes are opened together, how long will it take to fill the tank? Solution: Rate A = 1/6, Rate B = 1/8. Combined rate = 1/6 + 1/8 = 4/24 + 3/24 = 7/24. Time = 1 ÷ (7/24) = 24/7 = 3 3/7 hours. Answer: 24/7 hours = 3 3/7 hours
- Pipe A can fill a tank in 5 hours, while Pipe B (an outlet) can empty the full tank in 10 hours. If both pipes are opened together, how long will it take to fill the empty tank? Solution: Rate A = 1/5, Rate B = −1/10. Combined rate = 1/5 − 1/10 = 2/10 − 1/10 = 1/10. Time = 1 ÷ (1/10) = 10 hours. Answer: 10 hours

- Two pipes can fill a tank in 12 hours and 15 hours respectively. A third outlet pipe can empty the full tank in 20 hours. If all three are opened together, how long will it take to fill the empty tank? Solution: Rates: 1/12, 1/15, and −1/20. Using 60 as a common denominator: 5/60 + 4/60 − 3/60 = 6/60 = 1/10. Time = 1 ÷ (1/10) = 10 hours. Answer: 10 hours
- A tank can be filled by Pipe A in 4 hours. Pipe A is twice as fast as Pipe B. If both pipes work together, how long will they take to fill the tank? Solution: Pipe A's rate = 1/4. "A is twice as fast as B" means A's rate is double B's rate, so B's rate = (1/4) ÷ 2 = 1/8 (B alone would take 8 hours). Combined rate = 1/4 + 1/8 = 2/8 + 1/8 = 3/8. Time = 1 ÷ (3/8) = 8/3 = 2 2/3 hours. Answer: 8/3 hours = 2 2/3 hours
- Pipe A can fill a tank in 6 hours. When pipe A and an outlet pipe B are opened together, the tank fills in 10 hours. Find the time pipe B alone would take to empty the full tank. Solution: Set up the combined-rate equation with B's rate as the unknown: 1/6 − 1/y = 1/10. 1/y = 1/6 − 1/10 = 5/30 − 3/30 = 2/30 = 1/15. So y = 15 hours. Answer: 15 hours
- Pipe A can fill a tank in 12 hours, and Pipe B can fill it in 15 hours. Pipe A is opened first, and Pipe B joins 4 hours later. Find the total time taken to fill the tank. Solution: In the first 4 hours, only A works: it fills 4 × (1/12) = 1/3 of the tank. Remaining = 2/3. Once both are open, combined rate = 1/12 + 1/15 = 5/60 + 4/60 = 9/60 = 3/20. Time to fill the remaining 2/3 at this rate = (2/3) ÷ (3/20) = (2/3) × (20/3) = 40/9 hours. Total time = 4 + 40/9 = 36/9 + 40/9 = 76/9 ≈ 8.44 hours (8 hours and about 27 minutes). Answer: 76/9 hours ≈ 8.44 hours
- Pipes A and B can fill a tank in 15 and 20 hours respectively. Both are opened together, but after 4 hours, Pipe A is closed. Find the total time taken to fill the tank. Solution: Combined rate for the first 4 hours = 1/15 + 1/20 = 4/60 + 3/60 = 7/60. Filled in 4 hours = 4 × 7/60 = 28/60 = 7/15. Remaining = 8/15. Only B works after this, at rate 1/20. Time for the rest = (8/15) ÷ (1/20) = (8/15) × 20 = 160/15 = 32/3 hours ≈ 10.67 hours. Total time = 4 + 32/3 = 12/3 + 32/3 = 44/3 ≈ 14.67 hours (about 14 hours 40 minutes). Answer: 44/3 hours ≈ 14.67 hours
- Practice: Pipe A can fill a tank in 10 hours, and Pipe B can fill the same tank in 15 hours. If both are opened together, how long will it take to fill the tank? (Answer: 6 hours. Combined rate = 1/10 + 1/15 = 1/6.)
- Practice: Pipe A can fill a tank in 6 hours, while Pipe B (an outlet) can empty the full tank in 9 hours. If both are opened together, how long will it take to fill the empty tank? (Answer: 18 hours. Combined rate = 1/6 − 1/9 = 1/18.)
- Practice: A tank can be filled by Pipe A in 12 hours. Pipe A is 3 times as fast as Pipe B. If both work together, how long will they take to fill the tank? (Answer: 9 hours. B's rate = 1/36; combined = 1/12 + 1/36 = 1/9.)
- Practice: Pipe A can fill a tank in 8 hours. When pipe A and an outlet pipe B are opened together, the tank fills in 24 hours. Find the time pipe B alone would take to empty the tank. (Answer: 12 hours. 1/y = 1/8 − 1/24 = 1/12.)
- Practice: Pipe A can fill a tank in 12 hours, and Pipe B in 18 hours. Pipe A is opened first, and Pipe B joins 3 hours later. Find the total time to fill the tank. (Answer: 8.4 hours. First 3 hrs fill 1/4; remaining 3/4 at combined rate 5/36 takes 5.4 hrs; total 3+5.4.)
- Practice: Pipes A and B can fill a tank in 12 and 24 hours respectively. Both are opened together, but after 4 hours, Pipe A is closed. Find the total time to fill the tank. (Answer: 16 hours. First 4 hrs (combined rate 1/8) fill 1/2; remaining 1/2 at B's rate 1/24 takes 12 hrs; total 4+12.)
- Practice: A tank is filled by Pipe A in 9 hours. Pipe A is 1.5 times as fast as Pipe B. If both work together, how long will they take to fill the tank? (Answer: 5.4 hours. B's rate = (1/9)/1.5 = 2/27; combined = 3/27+2/27 = 5/27; time = 27/5.)

> **Quick revision:** A pipe that FILLS a tank has a positive rate (fraction of the tank filled per hour); a pipe that EMPTIES it (an outlet) has a negative rate. Combine rates exactly like Time and Work, but outlet pipes subtract; If a filling pipe can fill a tank in x hours, its rate is 1/x. If an outlet can empty a full tank in y hours, its rate is −1/y; With multiple pipes open together, add up all the rates (filling positive, emptying negative) to get the net rate. Time to fill (or empty) = 1 ÷ |net rate|, and the sign tells you which direction the tank is actually moving; A pipe that's "twice as fast" as another has half the time — rate and time are inverses here just like in Time and Work; Reverse problems (given the combined time, find one pipe's own time) just invert the usual setup: rearrange 1/x ± 1/y = 1/(combined time) algebraically for the unknown rate, then take its reciprocal to get that pipe's own time; If one pipe starts before another joins in, or a pipe is closed partway through, split the problem into stages exactly like in Time and Work: find the fraction of the tank filled in each stage, subtract running totals from 1, and solve the final stage with whichever pipe(s) remain

Work and Wages Solved Questions

When people work together and get paid as a group, the total wage is split in the same ratio as how much work each person actually did — which, for the same time worked, is the ratio of their individual work rates (1/days taken). This

page covers that rule, what changes when people work UNEQUAL numbers of days, and reverse problems, with several fully worked examples. If everyone works for the SAME number of days, wages are split in the ratio of their individual one-day work rates (1/days each person alone would take) — faster workers get a bigger share. Find each person's one-day work rate first (1/days alone), simplify the ratio between them, and only then divide the total wage in that ratio. For three or more people, the rate of the one not given directly can be found by subtracting the known rates from the combined rate — exactly like Time and Work. A larger wage share does not always mean more DAYS worked — it means a HIGHER work rate (faster individual completion time), since everyone here works the same number of days together. The "split by rate ratio" rule above only applies when everyone works the SAME number of days. If someone works fewer days than another (joins late, leaves early, or the days worked are simply stated as different), wages must be split by actual WORK DONE — each person's rate × the number of days THEY actually worked — not by the raw one-day rates alone. To handle this, compute each person's actual fraction of the job completed (rate × their own days worked), confirm these fractions add up to the whole job (1), and use those fractions — not the bare rates — as the wage-split ratio.

Examples

- A can complete a work in 10 days and B can complete the same work in 15 days. They work together and complete it, earning a total of ₹3000. Find A's share. Solution: A's one-day rate = 1/10, B's = 1/15. Ratio A : B = 1/10 : 1/15 = 15 : 10 = 3 : 2 (multiplying both by 30). A's share = 3/5 × ₹3000 = ₹1800. Answer: ₹1800
- Using the same data (A in 10 days, B in 15 days, total wage ₹3000), find B's share. Solution: From the same 3 : 2 ratio, B's share = 2/5 × ₹3000 = ₹1200. Answer: ₹1200
- A, B and C together can complete a work in 6 days. A alone can do it in 12 days, and B alone in 18 days. Working together, they earn ₹1800. Find C's share. Solution: Combined rate = 1/6. A's rate = 1/12, B's rate = 1/18. C's rate = 1/6 − 1/12 − 1/18. Using a common denominator of 36: 6/36 − 3/36 − 2/36 = 1/36. Ratio A : B : C = 3/36 : 2/36 : 1/36 = 3 : 2 : 1, total 6 parts. C's share = 1/6 × ₹1800 = ₹300. Answer: ₹300
- A and B together can complete a work in 12 days. A alone can do it in 20 days. Working together, they earn ₹2400. Find B's share. Solution: Combined rate = 1/12, A's rate = 1/20. B's rate = 1/12 − 1/20. Using a common denominator of 60: 5/60 − 3/60 = 2/60. Ratio A : B = 3/60 : 2/60 = 3 : 2, total 5 parts. B's share = 2/5 × ₹2400 = ₹960. Answer: ₹960
- A can complete a work in 10 days and B in 15 days. A works on the job for 4 days and B works for 9 days (not necessarily at the same time), and together they just complete the work. If the total wage is ₹2000, find each person's share based on actual work done. Solution: A's actual work = 4 × (1/10) = 2/5. B's actual work = 9 × (1/15) = 3/5. Check: 2/5 + 3/5 = 1, the whole job — consistent. Since the days worked are unequal, split by actual work done, not by the bare rates: ratio = 2/5 : 3/5 = 2 : 3. A's share = (2/5) × 2,000 = ₹800. B's share = (3/5) × 2,000 = ₹1,200. Answer: A = ₹800, B = ₹1,200
- A and B together can complete a work in 12 days. They work together for 4 days, after which A leaves and B completes the remaining work alone in 16 more days. If the total wage for the whole job is ₹4,800, find each person's share based on actual work done. Solution: Combined rate = 1/12. Work done in the first 4 days = 4/12 = 1/3, so the remaining work = 2/3. B alone finishes 2/3 in 16 days, so B's rate = (2/3)/16 = 1/24. Then A's rate = 1/12 − 1/24 = 1/24 (the same as B's, in this case). During the first 4 days, A contributed 4 × 1/24 = 1/6, and B also contributed 4 × 1/24 = 1/6. B then contributed a further 2/3 alone. B's total = 1/6 + 2/3 = 5/6. A's total = 1/6. (Check: 1/6 + 5/6 = 1.) Wage ratio A : B = 1/6 : 5/6 = 1 : 5. A's share = (1/6) × 4,800 = ₹800. B's share = (5/6) × 4,800 = ₹4,000. Answer: A = ₹800, B = ₹4,000
- A and B, working together for the same number of days, earn a total wage split in the ratio 3 : 2. If B can complete the work alone in 20 days, find how many days A would take alone. Solution: Since the wage ratio (with equal days worked) equals the rate ratio: rate A : rate B = 3 : 2. rate B = 1/20, so rate A = (3/2) × (1/20) = 3/40. A's time alone = 1 / (3/40) = 40/3 ≈ 13.3 days. Answer: 40/3 days ≈ 13.3 days
- Practice: A can complete a work in 8 days and B can complete it in 12 days. They work together and earn ₹2000 total. Find A's share. (Answer: ₹1200)
- Practice: Using the same data (A in 8 days, B in 12 days, total wage ₹2000), find B's share. (Answer: ₹800)
- Practice: A, B and C together can complete a work in 4 days. A alone takes 10 days and B alone takes 15 days. If they earn ₹1200 together, find C's share. (Answer: ₹400)
- Practice: A can complete a work in 12 days and B in 8 days. A works for 3 days and B works for 6 days, completing the work exactly. If the total wage is ₹1,400, find each person's share. (Answer: A = ₹350, B = ₹1,050. A's work = 3/12 =

1/4; B's work = 6/8 = 3/4; ratio 1:3.)

- Practice: A and B together can complete a work in 15 days. They work together for 5 days, after which A leaves and B completes the remaining work alone in 20 more days. If the total wage is ₹3,000, find each person's share. (Answer: A = ₹500, B = ₹2,500. Combined rate 1/15; both end up with equal individual rates of 1/30; A's total work = 1/6, B's = 5/6.)
- Practice: A and B, working together for the same number of days, earn a wage split in the ratio 2 : 3. If A can complete the work alone in 15 days, find how many days B would take alone. (Answer: 10 days. rate A = 1/15; rate B = (3/2) × (1/15) = 1/10.)
- Practice: A, B and C together can complete a work in 8 days. A alone takes 20 days and B alone takes 24 days. If they earn ₹1,440 together, find C's share. (Answer: ₹384. C's rate = 1/8−1/20−1/24 = 1/30; ratio A:B:C = 6:5:4; C's share = (4/15)×1,440.)

> **Quick revision:** If everyone works for the SAME number of days, wages are split in the ratio of their individual one-day work rates (1/days each person alone would take) — faster workers get a bigger share; Find each person's one-day work rate first (1/days alone), simplify the ratio between them, and only then divide the total wage in that ratio; For three or more people, the rate of the one not given directly can be found by subtracting the known rates from the combined rate — exactly like Time and Work; A larger wage share does not always mean more DAYS worked — it means a HIGHER work rate (faster individual completion time), since everyone here works the same number of days together; The "split by rate ratio" rule above only applies when everyone works the SAME number of days. If someone works fewer days than another (joins late, leaves early, or the days worked are simply stated as different), wages must be split by actual WORK DONE — each person's rate × the number of days THEY actually worked — not by the raw one-day rates alone; To handle this, compute each person's actual fraction of the job completed (rate × their own days worked), confirm these fractions add up to the whole job (1), and use those fractions — not the bare rates — as the wage-split ratio

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Time, Speed and Distance

Time, Speed and Distance — Concepts and Applications

Distance equals speed multiplied by time when the speed is constant, with consistent units throughout. Convert km/h to m/s by multiplying by 5/18 and reverse the conversion with 18/5. Average speed is total distance divided by total time, giving 2uv/(u + v) for equal distances at speeds u and v. Relative speed is the sum for opposite directions and the positive difference for the same direction. For trains use the total length that must clear the object, for boats combine still-water and stream speeds, and for races or circular tracks explicitly identify the distance gap that relative motion must close.

Trains crossing something need the FULL LENGTH of the train added to whatever it's crossing: crossing a stationary point object (a man, a pole) only needs the train's own length, but crossing a platform or bridge needs train length plus platform/bridge length, since the train isn't "clear" of the platform until its very last carriage passes the far end.

When two trains cross each other, the relevant speed is their relative speed (sum if moving in opposite directions, difference if the same direction, as with any relative-motion problem) and the relevant distance is the SUM of both train's lengths, since both trains must fully clear each other.

For races with a head start ("A gives B a start of X metres in a race of distance D"), the runners' speed ratio equals D : (D − X) — in the time A covers the full distance D, B only needs to cover D − X to finish level, so their speeds are in that same ratio.

Examples

- Convert 54 km/h to m/s. Use 1 km = 1000 m and 1 hour = 3600 seconds. Worked solution: 54 km/h = 54 × 1000/3600 m/s. Reducing the factor to 5/18 gives 15 m/s. Check: multiplying 15 by 3.6 returns 54 km/h.
- Two vehicles start simultaneously 276 km apart and drive directly towards each other at 41 and 51 km/h. Find the meeting time in hours. Worked solution: The gap closes at 41 + 51 = 92 km/h. Time = 276/92 = 3 hours. Check: their distances in 3 hours are 123 and 153 km, which add to the initial gap 276.

- Two runners start together at one point on a circular track of length 440 m. They run in the same direction at constant speeds 6 and 4 m/s. After how many seconds do they next meet? Worked solution: The faster runner must gain one complete lap of 440 m. Relative speed is 6 − 4 = 2 m/s, so time is 440/2 = 220 seconds. Check: in this time their travelled distances differ by exactly 440 m; no earlier positive time produces a full-lap difference.
- A train 150 m long crosses a 250 m platform at 72 km/h. Find the time taken. Worked solution: 72 km/h = 20 m/s; total distance to clear = 150 + 250 = 400 m; time = 400/20 = 20 seconds.
- A train 120 m long crosses a standing man in 8 seconds. Find the train's speed in km/h. Worked solution: Speed = 120/8 = 15 m/s = 15 × 18/5 = 54 km/h — only the train's own length matters when crossing a point object.
- Two trains 100 m and 150 m long move in opposite directions at 54 km/h and 36 km/h. Find the time they take to cross each other. Worked solution: Relative speed = (54+36) km/h = 90 km/h = 25 m/s; total length = 250 m; time = 250/25 = 10 seconds.
- A 100 m train at 72 km/h overtakes a 150 m train moving the same direction at 54 km/h. Find the time to cross. Worked solution: Relative speed = (72−54) km/h = 18 km/h = 5 m/s; total length = 250 m; time = 250/5 = 50 seconds.
- In a 100 m race, A can give B a head start of 20 m and they finish together. Find the ratio of their speeds. Worked solution: Speed ratio A:B = 100 : (100−20) = 100:80 = 5:4, since in the time A runs 100 m, B only needs to run 80 m to tie.

> **Quick revision:** Distance = speed × time; 1 m/s = 18/5 km/h; average speed = total distance/total time; use relative speed (sum for opposite, difference for same direction) for meetings and crossings. Trains crossing a point use only their own length; crossing a platform/bridge or another train adds both lengths. Race head-start ratio = full distance : (distance − head start).

Time, Speed and Distance Solved Questions

Time, Speed and Distance is one of the most frequently tested Quant topics across these exams — and most marks lost here come from a handful of recurring traps: forgetting to convert units, averaging two speeds the wrong way, and mixing up relative speed for opposite versus same-direction motion. This page covers the core rules, meeting-point and stoppage-time variants, and several fully worked examples. Speed = Distance / Time. Rearranged: Distance = Speed × Time, and Time = Distance / Speed. To convert km/hr to m/s, multiply by 5/18. To convert m/s to km/hr, multiply by 18/5. When the SAME distance is covered at two different speeds a and b (for example, going and returning by the same route), the average speed for the whole journey is 2ab/(a + b), not the simple average (a + b)/2. Using (a + b)/2 here is the single most common mistake in this topic — it only gives the right answer when the TIME spent at each speed is equal, not the distance. For a journey with different distances at different speeds (the equal-distance shortcut does not apply), fall back to the definition: average speed = total distance / total time. This always works, even when the shortcut above does not. For two objects moving toward each other, relative speed is the sum of their speeds; the time to meet (or to cross, for trains) uses this combined speed. For two objects moving in the same direction, relative speed is the difference of their speeds. Meeting-point problems: if two bodies start at the same time from two points a distance D apart and move toward each other at speeds a and b, they meet after time D/(a + b), at a distance a × D/(a + b) from the first point. Stoppage-time problems: if a train's speed excluding stoppages is s_1 and its effective (average, including stoppages) speed is s_2, the time lost to stoppages per hour of travel = $(1 - s_2/s_1) \times 60$ minutes. Keep units consistent throughout a single calculation — mixing km with minutes, or m/s with hours, without converting first is a common source of wrong answers even when the method is right.

Examples

- A car travels at a constant speed of 60 km/hr. How long will it take to cover 180 km? Solution: Time = Distance / Speed = 180 / 60 = 3 hours Answer: 3 hours
- Convert a speed of 72 km/hr into metres per second. Solution: To convert km/hr to m/s, multiply by 5/18. 72 × 5/18 = 360/18 = 20 m/s Answer: 20 m/s
- A man travels from town A to town B at 40 km/hr, and returns from B to A by the same road at 60 km/hr. Find his average speed for the entire journey. Solution: The distance each way is the same, so the simple average (40 + 60)/2 = 50 km/hr is WRONG here — that shortcut only works when the TIME spent at each speed is equal, not the distance. For

two equal distances covered at speeds a and b, average speed = 2ab / (a + b). = (2 × 40 × 60) / (40 + 60) = 4,800 / 100 = 48 km/hr Answer: 48 km/hr

- A man travels 60 km at 30 km/hr, and then a further 90 km at 45 km/hr. Find his average speed for the whole journey. Solution: The two distances (60 km and 90 km) are not equal, so the 2ab/(a + b) shortcut does not apply here — use total distance / total time instead. Time for the first part = 60/30 = 2 hours. Time for the second part = 90/45 = 2 hours. Total distance = 60 + 90 = 150 km. Total time = 2 + 2 = 4 hours. Average speed = 150 / 4 = 37.5 km/hr. Answer: 37.5 km/hr
- Two trains, 150 m and 100 m long, run on parallel tracks in opposite directions at 45 km/hr and 36 km/hr respectively. In how much time will they completely cross each other? Solution: Moving in opposite directions, relative speed = sum of speeds = 45 + 36 = 81 km/hr. Convert to m/s: 81 × 5/18 = 22.5 m/s. To completely cross each other, together they must cover a distance equal to the sum of both train lengths: 150 + 100 = 250 m. Time = Distance / Speed = 250 / 22.5 = 100/9 ≈ 11.1 seconds. Answer: 100/9 seconds ≈ 11.1 seconds
- Towns A and B are 300 km apart. A car starts from A towards B at 50 km/hr at the same time as another car starts from B towards A at 70 km/hr. After how long do they meet, and how far from A? Solution: Moving toward each other, combined speed = 50 + 70 = 120 km/hr. Time to meet = Distance / combined speed = 300 / 120 = 2.5 hours. Distance from A = speed from A × time = 50 × 2.5 = 125 km. Answer: They meet after 2.5 hours, 125 km from A
- A train's speed excluding stoppages is 60 km/hr, and its average speed including stoppages is 45 km/hr. Find the number of minutes per hour the train is stopped. Solution: Fraction of each hour "lost" to stoppages = 1 − (effective speed / running speed) = 1 − 45/60 = 1 − 0.75 = 0.25. Stoppage time per hour = 0.25 × 60 minutes = 15 minutes Answer: 15 minutes per hour
- Practice: A cyclist covers 150 km in 6 hours. Find his speed in km/hr. (Answer: 25 km/hr. Speed = 150/6.)
- Practice: Convert a speed of 90 km/hr into metres per second. (Answer: 25 m/s. 90 × 5/18 = 25.)
- Practice: A train covers a certain distance at 50 km/hr and returns over the same route at 75 km/hr. Find its average speed for the whole trip. (Answer: 60 km/hr. 2ab/(a+b) = (2×50×75)/125 = 7,500/125.)
- Practice: A man walks 20 km at 4 km/hr and then cycles 60 km at 20 km/hr. Find his average speed for the whole journey. (Answer: 10 km/hr. Time = 20/4 + 60/20 = 5 + 3 = 8 hours; total distance = 80 km; average = 80/8.)
- Practice: Two cyclists start at the same time from points 72 km apart and ride towards each other at 15 km/hr and 21 km/hr. After how long do they meet? (Answer: 2 hours. 72/(15+21) = 72/36.)
- Practice: A bus's speed excluding stoppages is 50 km/hr, and including stoppages it is 40 km/hr. Find the stoppage time per hour. (Answer: 12 minutes. (1 − 40/50) × 60 = 0.2 × 60.)
- Practice: Two trains 120 m and 180 m long move in the same direction at 72 km/hr and 54 km/hr. Find the time the faster train takes to completely pass the slower one. (Answer: 60 seconds. Relative speed = 72−54 = 18 km/hr = 5 m/s; distance = 120+180 = 300 m; time = 300/5.)

Quick revision: Speed = Distance / Time. Rearranged: Distance = Speed × Time, and Time = Distance / Speed; To convert km/hr to m/s, multiply by 5/18. To convert m/s to km/hr, multiply by 18/5; When the SAME distance is covered at two different speeds a and b (for example, going and returning by the same route), the average speed for the whole journey is 2ab/(a + b), not the simple average (a + b)/2. Using (a + b)/2 here is the single most common mistake in this topic — it only gives the right answer when the TIME spent at each speed is equal, not the distance; For a journey with different distances at different speeds (the equal-distance shortcut does not apply), fall back to the definition: average speed = total distance / total time. This always works, even when the shortcut above does not; For two objects moving toward each other, relative speed is the sum of their speeds; the time to meet (or to cross, for trains) uses this combined speed. For two objects moving in the same direction, relative speed is the difference of their speeds; Meeting-point problems: if two bodies start at the same time from two points a distance D apart and move toward each other at speeds a and b, they meet after time D/(a + b), at a distance a × D/(a + b) from the first point; Stoppage-time problems: if a train's speed excluding stoppages is s_1 and its effective (average, including stoppages) speed is s_2, the time lost to stoppages per hour of travel = $(1 - s_2/s_1) \times 60$ minutes; Keep units consistent throughout a single calculation — mixing km with minutes, or m/s with hours, without converting first is a common source of wrong answers even when the method is right

Problems on Trains Solved Questions

Train problems are Time-Speed-Distance with one extra twist: the train itself has a length, so "crossing" something means covering that length too — a pole, a platform, a walking person, or another train. This page covers all four standard scenarios, a walking person moving in either direction, and the "meet, then continue" ratio shortcut, with several fully worked examples. Crossing a stationary point (a pole, a man standing still, a signal) means the train covers a distance equal to its OWN length — treat the point as having zero length. Crossing a platform (or bridge) means the train covers a distance equal to its own length PLUS the platform's length — the whole train must clear the whole platform. Two trains crossing each other: use relative speed (sum of speeds if opposite directions, difference if same direction), and the distance to cover is the SUM of both train lengths — the same rule as objects, just with two lengths now. If the train crosses a person who is also moving (walking, not standing still), use relative speed exactly as with two trains — add the speeds for opposite directions, subtract for the same direction — but the distance to cover is still just the train's own length, since a person has no length to add. "Meet, then continue" problems: if two trains start towards each other and, after crossing, take a and b hours respectively to reach the OTHER train's starting point, then (speed of train 1) / (speed of train 2) = $\sqrt{(b/a)}$. Always convert km/hr to m/s (× 5/18) before working with lengths in metres and time in seconds — mixing units is the most common mistake here.

Examples

- A train 180 m long is running at 54 km/hr. How long will it take to cross a man standing on the platform? Solution: Convert speed to m/s: 54 × 5/18 = 15 m/s. Crossing a man (a point) means covering a distance equal to the train's own length: 180 m. Time = Distance / Speed = 180 / 15 = 12 seconds. Answer: 12 seconds
- A train 150 m long running at 72 km/hr crosses a platform in 25 seconds. Find the length of the platform. Solution: Convert speed to m/s: 72 × 5/18 = 20 m/s. Distance covered in 25 seconds = 20 × 25 = 500 m. This distance is the train's length plus the platform's length, so platform length = 500 − 150 = 350 m. Answer: 350 m
- Two trains, 120 m and 180 m long, run on parallel tracks in the SAME direction at 72 km/hr and 54 km/hr. Find the time for the faster train to completely cross the slower one. Solution: Same direction, so relative speed = difference of speeds = 72 − 54 = 18 km/hr. Convert to m/s: 18 × 5/18 = 5 m/s. Distance to cover = sum of both lengths = 120 + 180 = 300 m. Time = Distance / Speed = 300 / 5 = 60 seconds. Answer: 60 seconds
- Two trains, 140 m and 160 m long, run on parallel tracks in OPPOSITE directions at 60 km/hr and 48 km/hr. Find the time for them to cross each other. Solution: Opposite directions, so relative speed = sum of speeds = 60 + 48 = 108 km/hr. Convert to m/s: 108 × 5/18 = 30 m/s. Distance to cover = sum of both lengths = 140 + 160 = 300 m. Time = Distance / Speed = 300 / 30 = 10 seconds. Answer: 10 seconds
- A train 200 m long is running at 66 km/hr. It crosses a man walking in the OPPOSITE direction at 6 km/hr. Find the time taken to cross him. Solution: Opposite directions, so relative speed = 66 + 6 = 72 km/hr. Convert to m/s: 72 × 5/18 = 20 m/s. Distance to cover is just the train's length (the man has no length): 200 m. Time = 200 / 20 = 10 seconds. Answer: 10 seconds
- A train 150 m long running at 70 km/hr crosses a man walking in the SAME direction at 10 km/hr. Find the time taken to cross him. Solution: Same direction, so relative speed = 70 − 10 = 60 km/hr. Convert to m/s: 60 × 5/18 = 50/3 m/s. Time = Distance / Speed = 150 / (50/3) = 150 × 3/50 = 9 seconds. Answer: 9 seconds
- Two trains start at the same time from stations A and B towards each other. After crossing each other, they take 4 hours and 9 hours respectively to reach stations B and A. Find the ratio of their speeds. Solution: Using the shortcut: speed of train from A / speed of train from B = $\sqrt{(b/a)}$, where a = 4 (A-train's remaining time) and b = 9 (B-train's remaining time). = $\sqrt{(9/4)}$ = 3/2. So the speeds are in the ratio 3 : 2 — the train from A is faster, consistent with it needing less time to finish its remaining distance. Answer: 3 : 2
- Practice: A train 200 m long is running at 90 km/hr. How long will it take to cross a lamp post? (Answer: 8 seconds)
- Practice: A train 250 m long running at 108 km/hr crosses a platform in 30 seconds. Find the length of the platform. (Answer: 650 m)
- Practice: Two trains, 100 m and 140 m long, run on parallel tracks in the same direction at 63 km/hr and 45 km/hr. Find the time for the faster train to completely cross the slower one. (Answer: 48 seconds)
- Practice: A train 180 m long is running at 50 km/hr. It crosses a man walking in the opposite direction at 4 km/hr. Find the time taken. (Answer: 12 seconds. Relative speed = 54 km/hr = 15 m/s; 180/15.)

- Practice: A train 120 m long running at 56 km/hr crosses a man walking in the same direction at 8 km/hr. Find the time taken. (Answer: 9 seconds. Relative speed = 48 km/hr = 40/3 m/s; 120 ÷ (40/3).)
- Practice: Two trains start simultaneously from stations P and Q towards each other. After crossing, they take 16 hours and 9 hours respectively to reach Q and P. Find the ratio of their speeds. (Answer: 3 : 4. √(9/16) = 3/4.)
- Practice: A train crosses a man standing on a platform in 10 seconds, and crosses the entire 300 m long platform in 25 seconds. Find the length and speed of the train. (Answer: Length = 200 m, Speed = 20 m/s. Let speed = s; train length = 10s; 10s + 300 = 25s gives s = 20.)

> **Quick revision:** Crossing a stationary point (a pole, a man standing still, a signal) means the train covers a distance equal to its OWN length — treat the point as having zero length; Crossing a platform (or bridge) means the train covers a distance equal to its own length PLUS the platform's length — the whole train must clear the whole platform; Two trains crossing each other: use relative speed (sum of speeds if opposite directions, difference if same direction), and the distance to cover is the SUM of both train lengths — the same rule as objects, just with two lengths now; If the train crosses a person who is also moving (walking, not standing still), use relative speed exactly as with two trains — add the speeds for opposite directions, subtract for the same direction — but the distance to cover is still just the train's own length, since a person has no length to add; "Meet, then continue" problems: if two trains start towards each other and, after crossing, take a and b hours respectively to reach the OTHER train's starting point, then (speed of train 1) / (speed of train 2) = √(b/a); Always convert km/hr to m/s (× 5/18) before working with lengths in metres and time in seconds — mixing units is the most common mistake here

Races and Games Solved Questions

A race is just Time-Speed-Distance with a finish line — the twist is "start" (a head start in distance or time) and "beats by" (the gap at the finish), both of which just mean comparing how far each runner has gone at a shared moment in time. This page covers those standard patterns, converting between a distance start and a time start, combining three runners' results, and the "wins by X m or Y seconds" composite question, with several fully worked examples. "A beats B by X metres" means: at the moment A finishes, B still has X metres left to run. "A gives B a start of X metres" means B only has to cover (total distance – X) to finish — B effectively starts X metres ahead. "A gives B a start of X seconds" means B starts running X seconds before A does — add X seconds to A's finishing time to get B's head-start-adjusted clock. For a "dead heat" (both finish together), set up the condition that both cover their required distance in exactly the same time, then solve for the unknown. A "distance start" and a "time start" convert into each other once you know the slower runner's speed: a start of T seconds is equivalent to a start of (T × that runner's speed) metres, and vice versa. For three runners, where you're told how much A beats B by and how much B beats C by (as a fraction or percentage of the race distance), combine the two speed RATIOS by multiplying them — don't just add the two "beats by" distances, since each one is measured against a different base.

Examples

- A and B run a 100 m race. A finishes in 10 seconds, and B finishes in 12.5 seconds (both at constant speed). By what distance does A beat B? Solution: B's speed = 100 / 12.5 = 8 m/s. At t = 10 seconds (when A finishes), B has covered 8 × 10 = 80 m. A beats B by 100 – 80 = 20 m. Answer: 20 m
- In a 100 m race, A gives B a start of 20 m. A's speed is 10 m/s and B's speed is 5 m/s. By how much time does A beat B? Solution: A runs the full 100 m: time = 100 / 10 = 10 seconds. B only needs to cover 100 – 20 = 80 m: time = 80 / 5 = 16 seconds. A beats B by 16 – 10 = 6 seconds. Answer: 6 seconds
- In a 200 m race, A gives B a start of 4 seconds and still beats him by 20 m. If A's speed is 10 m/s, find B's speed. Solution: A's time for 200 m = 200 / 10 = 20 seconds. B started 4 seconds earlier, so B has been running for 20 + 4 = 24 seconds when A finishes. A beats B by 20 m, so in those 24 seconds B covered 200 – 20 = 180 m. B's speed = 180 / 24 = 7.5 m/s. Answer: 7.5 m/s
- In a 200 m race, A's speed is 10 m/s and B's speed is 8 m/s. What start should A give B so that the race ends in a dead heat (both finish together)? Solution: A's time to finish 200 m = 200 / 10 = 20 seconds. In that same 20 seconds, B covers 8 × 20 = 160 m. For a dead heat, B's required distance must also be 160 m, so the start = 200 – 160 = 40 m. Answer: 40 m

- In a 100 m race, A gives B a start of 5 seconds. If B's speed is 4 m/s, what is this equivalent to as a distance start? Solution: A time start of T seconds is equivalent to that runner covering T × speed metres before the other even begins. Equivalent distance = 5 × 4 = 20 m. Answer: 20 m
- In a race, A can give B a start of 10 m in a 100 m race, and B can give C a start of 10 m in the same 100 m race. If A, B and C all ran a 100 m race together, by how much would A beat C? Solution: A gives B a 10 m start in 100 m, so while A covers 100 m, B covers 90 m: speed ratio B/A = 90/100 = 9/10. Similarly, speed ratio C/B = 9/10. Combine the ratios (don't just add the distances): speed ratio C/A = (9/10) × (9/10) = 81/100. So while A covers 100 m, C covers 81 m. A beats C by 100 − 81 = 19 m (not 10 + 10 = 20 m). Answer: 19 m
- A and B run a 400 m race. A wins by 40 m or 10 seconds. Find A's speed. Solution: The "40 m or 10 seconds" phrasing means: the 40 m that B was still short of finishing is exactly what B would have covered in the 10 extra seconds. So B's speed = 40/10 = 4 m/s. B's time to finish the full 400 m = 400/4 = 100 seconds. A finishes 10 seconds before that: A's time = 100 − 10 = 90 seconds. A's speed = 400/90 = 40/9 ≈ 4.44 m/s. Answer: 40/9 m/s ≈ 4.44 m/s
- Practice: A and B run a 200 m race. A finishes in 20 seconds, and B finishes in 25 seconds (both at constant speed). By what distance does A beat B? (Answer: 40 m)
- Practice: In a 150 m race, A gives B a start of 30 m. A's speed is 15 m/s and B's speed is 10 m/s. By how much time does A beat B? (Answer: 2 seconds)
- Practice: In a 300 m race, A's speed is 12 m/s and B's speed is 9 m/s. What start should A give B so that the race ends in a dead heat? (Answer: 75 m)
- Practice: In a 200 m race, A gives B a start of 8 seconds. If B's speed is 5 m/s, what is this equivalent to as a distance start? (Answer: 40 m. 8 × 5.)
- Practice: In a race, A can give B a start of 20 m in a 100 m race, and B can give C a start of 10 m in the same 100 m race. By how much would A beat C in a 100 m race among all three? (Answer: 28 m. Speed ratio C/A = (80/100) × (90/100) = 18/25; C covers 72 m while A covers 100 m.)
- Practice: A and B run a 500 m race. A wins by 50 m or 10 seconds. Find A's speed. (Answer: 50/9 m/s ≈ 5.56 m/s. B's speed = 50/10 = 5 m/s; B's time = 100 s; A's time = 90 s; A's speed = 500/90.)
- Practice: In a 250 m race, A's speed is 10 m/s and B's speed is 8 m/s. What start should A give B for a dead heat? (Answer: 50 m. A's time = 25 s; B covers 200 m in 25 s; start = 250 − 200.)

> **Quick revision:** "A beats B by X metres" means: at the moment A finishes, B still has X metres left to run; "A gives B a start of X metres" means B only has to cover (total distance − X) to finish — B effectively starts X metres ahead; "A gives B a start of X seconds" means B starts running X seconds before A does — add X seconds to A's finishing time to get B's head-start-adjusted clock; For a "dead heat" (both finish together), set up the condition that both cover their required distance in exactly the same time, then solve for the unknown; A "distance start" and a "time start" convert into each other once you know the slower runner's speed: a start of T seconds is equivalent to a start of (T × that runner's speed) metres, and vice versa; For three runners, where you're told how much A beats B by and how much B beats C by (as a fraction or percentage of the race distance), combine the two speed RATIOS by multiplying them — don't just add the two "beats by" distances, since each one is measured against a different base

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Algebra

Algebra — Concepts and Applications

Use $(a + b)^2 = a^2 + 2ab + b^2$, $(a - b)^2 = a^2 - 2ab + b^2$ and $(a + b)(a - b) = a^2 - b^2$ to expand or factor expressions. Factorisation rewrites an expression as a product, and cancellation of a factor is valid only when that factor is nonzero. Substitute given values with brackets around negative numbers and preserve the order of operations. For non-negative values, extract square factors from square roots, and rationalise a binomial surd denominator by multiplying numerator and denominator by its conjugate. Solve linear equations by applying the same reversible operation to both sides; the graph of $y = mx + c$ is a straight line with slope m and y-intercept c.

A quadratic equation $ax^2 + bx + c = 0$ can often be solved by factoring it into two linear brackets whose product is zero - if it factors as $(x - p)(x - q) = 0$, then $x = p$ or $x = q$, since a product is zero only when at least one factor is

zero. When factoring isn't obvious, the quadratic formula $x = [-b \pm \sqrt{(b^2-4ac)}] / 2a$ always works.

For a pair of simultaneous linear equations in two variables, the elimination method (multiply one or both equations so that one variable's coefficients match, then add or subtract to cancel it) reduces the problem to a single-variable equation, which can then be substituted back to find the other variable.

The cube identities extend the same pattern: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ and $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ - useful both for factoring cubic-looking expressions and for computing sums/differences of cubes without cubing large numbers directly.

Examples

- Evaluate $(21 + 4)^2 - (21 - 4)^2$. Worked solution: Expand both squares. The a^2 and b^2 terms cancel, leaving $4ab = 4 \times 21 \times 4 = 336$. Check directly: $25^2 - 17^2 = 625 - 289 = 336$.
- The principal square root $\sqrt{27}$ is written as $k\sqrt{3}$ with k positive. Find k. Worked solution: $27 = 9 \times 3$. Taking the principal square root gives $\sqrt{9} \times \sqrt{3} = 3\sqrt{3}$, so $k = 3$. Check: $(3\sqrt{3})^2 = 9 \times 3 = 27$; the coefficient is positive as required.
- The line $y = 2x - 8$ meets the x-axis. Find the x-coordinate of this intercept. Worked solution: Every point on the x-axis has $y = 0$. Set $0 = 2x - 8$, giving $x = 8/2 = 4$. Check: at $x = 4$, the equation gives $y = 0$, so the point lies on the x-axis.
- Solve $x^2 - 5x + 6 = 0$ by factoring. Worked solution: Looking for two numbers that multiply to 6 and add to -5 gives -2 and -3, so the equation factors as $(x-2)(x-3) = 0$, giving $x = 2$ or $x = 3$. Check: $2^2-5\times2+6 = 4-10+6 = 0$, and $3^2-5\times3+6 = 9-15+6 = 0$.
- Solve the simultaneous equations $2x + 3y = 19$ and $x - y = 2$. Worked solution: From the second equation, $x = y + 2$. Substituting: $2(y+2) + 3y = 19 \rightarrow 5y + 4 = 19 \rightarrow y = 3$, so $x = 5$. Check: $2(5)+3(3) = 10+9 = 19$ ✓ and $5-3 = 2$ ✓.
- Find $3^3 + 4^3$ using the sum-of-cubes identity. Worked solution: $a^3+b^3 = (a+b)(a^2-ab+b^2) = (3+4)(9-12+16) = 7\times13 = 91$. Check directly: $27 + 64 = 91$.
- Rationalise the denominator of $1/(\sqrt{5} - \sqrt{3})$. Worked solution: Multiply numerator and denominator by the conjugate $(\sqrt{5}+\sqrt{3})$: $(\sqrt{5}+\sqrt{3})/((\sqrt{5})^2-(\sqrt{3})^2) = (\sqrt{5}+\sqrt{3})/(5-3) = (\sqrt{5}+\sqrt{3})/2$.

> **Quick revision:** $(a\pm b)^2 = a^2\pm2ab+b^2$; $(a+b)(a-b)=a^2-b^2$; cancel factors only when nonzero; rationalise with conjugates; $y=mx+c$ has slope m. Quadratics: factor into $(x-p)(x-q)=0$ or use the quadratic formula. Simultaneous equations: eliminate one variable by matching coefficients. Cube identities: $a^3\pm b^3 = (a\pm b)(a^2\mp ab+b^2)$.

Surds and Indices Solved Questions

A handful of fixed rules for combining powers of the same base cover almost everything in this topic — the only real skill is applying them in the right order. This page covers those rules, fractional exponents, solving same-base equations, and comparing surds, with several fully worked examples. For powers of the SAME base: $a^m \times a^n = a^{m+n}$ (add exponents when multiplying); $a^m \div a^n = a^{m-n}$ (subtract when dividing); $(a^m)^n = a^{mn}$ (multiply when raising a power to a power). Any nonzero number to the power 0 equals 1 ($a^0 = 1$). A negative exponent means "reciprocal": $a^{-n} = 1/a^n$. A surd is an irrational root that can't be simplified to a whole number, like $\sqrt{2}$ or $\sqrt{3}$ ($\sqrt{4} = 2$ is NOT a surd, since it simplifies exactly). To simplify one, pull out the largest perfect-square factor: $\sqrt{50} = \sqrt{(25 \times 2)} = 5\sqrt{2}$. To "rationalize" a fraction with a surd in the denominator, multiply top and bottom by that surd (or its conjugate, if the denominator has two terms), so the denominator becomes a whole number: $1/\sqrt{2} = \sqrt{2}/2$. Fractional exponents are roots: a^(1/n) = $\sqrt[n]{a}$ (the nth root of a), and a^(m/n) = ($\sqrt[n]{a}$)^m — apply the root and the power in either order, whichever is easier to compute by hand. To solve an equation where both sides are powers of the same base, equate just the exponents: if a^x = a^y (with $a > 0$, $a \neq 1$), then $x = y$. Only "like" surds (the same number under the root) combine directly, by adding or subtracting their coefficients — $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$, but $\sqrt{2} + \sqrt{3}$ cannot be merged into a single surd term. To compare two different surds, square both sides to remove the root, then compare the squared values.

Examples

- Simplify: $2^5 \times 2^3$ Solution: Same base, so add the exponents: $2^{5+3} = 2^8$. $2^8 = 256$. Answer: 256
- Simplify: $(3^4)^2 \div 3^5$ Solution: $(3^4)^2 = 3^8$ (multiply the exponents). $3^8 \div 3^5 = 3^{8-5} = 3^3 = 27$. Answer: 27
- Simplify $\sqrt{72}$. Solution: $72 = 36 \times 2$, and 36 is the largest perfect-square factor of 72. $\sqrt{72} = \sqrt{(36 \times 2)} = 6\sqrt{2}$. Answer: $6\sqrt{2}$
- Rationalize: $1 / (2 + \sqrt{3})$ Solution: Multiply top and bottom by the conjugate, $(2 - \sqrt{3})$: $[1 \times (2 - \sqrt{3})] / [(2 + \sqrt{3})(2 - \sqrt{3})]$. The denominator becomes $2^2 - (\sqrt{3})^2 = 4 - 3 = 1$. Result: $(2 - \sqrt{3}) / 1 = 2 - \sqrt{3}$. Answer: $2 - \sqrt{3}$

- Simplify: 8^(2/3) Solution: 8^(2/3) = (cube root of 8)², since the denominator of the fractional exponent is the root. Cube root of 8 = 2, so the result is $2^2 = 4$. Answer: 4
- Solve for x: 2^(x+1) = 32 Solution: Write 32 as a power of 2: $32 = 2^5$. Now both sides have the same base, so equate the exponents: x + 1 = 5. x = 4. Answer: x = 4
- Simplify: 3√5 + 2√5 − √5 Solution: All three terms are like surds (same number, 5, under the root), so combine their coefficients. (3 + 2 − 1)√5 = 4√5. Answer: 4√5
- Which is greater: 2√5 or 3√3? Solution: Square both to remove the roots: $(2\sqrt{5})^2 = 4 \times 5 = 20$. $(3\sqrt{3})^2 = 9 \times 3 = 27$. Since 27 > 20, 3√3 is greater. Answer: 3√3 is greater
- Practice: Simplify: $5^6 \div 5^2$ (Answer: 625)
- Practice: Simplify √98. (Answer: 7√2)
- Practice: Simplify: $(2^3)^2 \times 2^{-4}$ (Answer: 4)
- Practice: Simplify: 27^(2/3) (Answer: 9. Cube root of 27 is 3, then $3^2 = 9$.)
- Practice: Solve for x: 3^(2x−1) = 243 (Answer: x = 3. $243 = 3^5$, so 2x−1 = 5.)
- Practice: Simplify: 4√3 + 5√3 − 2√3 (Answer: 7√3. Like surds: (4+5−2)√3.)
- Practice: Which is greater: 4√2 or 3√3? (Answer: 4√2 is greater. $(4\sqrt{2})^2 = 32$ and $(3\sqrt{3})^2 = 27$; 32 > 27.)

Quick revision: For powers of the SAME base: $a^m \times a^n = a^{m+n}$ (add exponents when multiplying); $a^m \div a^n = a^{m-n}$ (subtract when dividing); $(a^m)^n = a^{mn}$ (multiply when raising a power to a power); Any nonzero number to the power 0 equals 1 ($a^0 = 1$). A negative exponent means "reciprocal": $a^{-n} = 1/a^n$; A surd is an irrational root that can't be simplified to a whole number, like √2 or √3 (√4 = 2 is NOT a surd, since it simplifies exactly). To simplify one, pull out the largest perfect-square factor: √50 = √(25 × 2) = 5√2; To "rationalize" a fraction with a surd in the denominator, multiply top and bottom by that surd (or its conjugate, if the denominator has two terms), so the denominator becomes a whole number: 1/√2 = √2/2; Fractional exponents are roots: a^(1/n) = $\sqrt[n]{a}$ (the nth root of a), and a^(m/n) = ($\sqrt[n]{a}$)^m — apply the root and the power in either order, whichever is easier to compute by hand; To solve an equation where both sides are powers of the same base, equate just the exponents: if a^x = a^y (with a > 0, a ≠ 1), then x = y; Only "like" surds (the same number under the root) combine directly, by adding or subtracting their coefficients — 3√2 + 5√2 = 8√2, but √2 + √3 cannot be merged into a single surd term. To compare two different surds, square both sides to remove the root, then compare the squared values

Quadratic Equations Solved Questions

This topic isn't about solving one quadratic equation — it's about solving two of them (one in x, one in y) and then comparing every possible pair of roots to decide the relationship between x and y. This page covers that approach, the quadratic formula and discriminant for equations that don't factor neatly, and the sum/product-of-roots shortcut, with several fully worked examples. Solve equation I for all values of x, and equation II for all values of y — each quadratic usually gives two roots. Compare every x with every y. If x is bigger in every single combination, the answer is "x > y"; if x is always smaller, it's "x < y". If some combinations give x bigger and others give x equal, the answer is "x ≥ y" (and symmetrically "x ≤ y") — not "x > y", since equality also occurs. If the combinations genuinely conflict — some give x > y, others x < y (or x = y) — no single relationship covers every case, so the answer is "cannot be determined". When an equation doesn't factor neatly into integers, use the quadratic formula: for $ax^2 + bx + c = 0$, $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$. The discriminant $D = b^2 - 4ac$ tells you the nature of the roots before you even finish solving: D > 0 gives two distinct real roots, D = 0 gives one repeated real root, D < 0 gives no real roots at all. Shortcut without fully solving: sum of roots = −b/a, and product of roots = c/a. A negative product means the two roots have opposite signs (one positive, one negative) — worth checking early, since a negative root often breaks what would otherwise look like a clean "x > y" or "x < y" pattern. Common mistakes: dropping the ± when taking a square root in the quadratic formula, and sign slips while factoring — always re-expand a factored form mentally to confirm it reproduces the original equation.

Examples

- I. $x^2 - 11x + 30 = 0$ II. $y^2 - 9y + 20 = 0$. Find the relationship between x and y. Solution: I: $x^2 - 11x + 30 = (x - 5)(x - 6) = 0$, so x = 5 or 6. II: $y^2 - 9y + 20 = (y - 4)(y - 5) = 0$, so y = 4 or 5. Comparing every pair: (5,4)→x>y, (5,5)→x=y, (6,4)→x>y, (6,5)→x>y. x is never smaller than y, but can equal it — so the relationship is x ≥ y. Answer: x ≥ y

- I. $x^2 - 9x + 20 = 0$ II. $y^2 - 11y + 30 = 0$. Find the relationship between x and y. Solution: I: $x^2 - 9x + 20 = (x - 4)(x - 5) = 0$, so x = 4 or 5. II: $y^2 - 11y + 30 = (y - 5)(y - 6) = 0$, so y = 5 or 6. Comparing every pair: (4,5)→x<y, (4,6)→x<y, (5,5)→x=y, (5,6)→x<y. x is never bigger than y, but can equal it — so the relationship is $x \le y$. Answer: $x \le y$
- I. $x^2 - 7x + 12 = 0$ II. $y^2 - 3y + 2 = 0$. Find the relationship between x and y. Solution: I: $x^2 - 7x + 12 = (x - 3)(x - 4) = 0$, so x = 3 or 4. II: $y^2 - 3y + 2 = (y - 1)(y - 2) = 0$, so y = 1 or 2. Every value of x (3 or 4) is bigger than every value of y (1 or 2) — all four combinations give x>y with no equality possible. Answer: $x > y$
- I. $x^2 - 3x + 2 = 0$ II. $y^2 - 7y + 12 = 0$. Find the relationship between x and y. Solution: I: $x^2 - 3x + 2 = (x - 1)(x - 2) = 0$, so x = 1 or 2. II: $y^2 - 7y + 12 = (y - 3)(y - 4) = 0$, so y = 3 or 4. Every value of x (1 or 2) is smaller than every value of y (3 or 4) — all four combinations give x<y with no equality possible. Answer: $x < y$
- Solve $x^2 - 4x - 1 = 0$ using the quadratic formula. Solution: Here a=1, b=−4, c=−1. Discriminant $D = b^2 - 4ac = 16 - 4(1)(-1) = 16 + 4 = 20$. $x = [-b \pm \sqrt{D}] / 2a = [4 \pm \sqrt{20}] / 2 = [4 \pm 2\sqrt{5}] / 2 = 2 \pm \sqrt{5}$. So $x = 2 + \sqrt{5} \approx 4.24$, or $x = 2 - \sqrt{5} \approx -0.24$. Answer: $x = 2 + \sqrt{5}$ or $2 - \sqrt{5}$ (≈ 4.24 or ≈ -0.24)
- I. $x^2 - 8x + 15 = 0$ II. $y^2 - 2y - 15 = 0$. Find the relationship between x and y. Solution: I: $x^2 - 8x + 15 = (x - 3)(x - 5) = 0$, so x = 3 or 5. II: $y^2 - 2y - 15 = (y - 5)(y + 3) = 0$, so y = 5 or −3. The negative product of roots (−15) for equation II was a hint that one root would be negative. Comparing every pair: (3,5)→x<y, (3,−3)→x>y, (5,5)→x=y, (5,−3)→x>y. The results genuinely conflict (both > and < occur), so the relationship cannot be determined. Answer: Cannot be determined
- Determine the nature of the roots of $x^2 + 2x + 5 = 0$, without fully solving it. Solution: a=1, b=2, c=5. Discriminant $D = b^2 - 4ac = 4 - 20 = -16$. Since $D < 0$, the equation has no real roots (its roots are complex). Answer: No real roots ($D = -16 < 0$)
- Practice: I. $x^2 - 6x + 9 = 0$ II. $y^2 - 6y + 9 = 0$. Find the relationship between x and y. (Answer: x = y)
- Practice: I. $x^2 - 5x + 6 = 0$ II. $y^2 - 4y + 3 = 0$. Find the relationship between x and y. (Answer: Cannot be determined)
- Practice: I. $x^2 - 4x + 3 = 0$ II. $y^2 - 11y + 30 = 0$. Find the relationship between x and y. (Answer: x < y)
- Practice: Solve $x^2 - 6x - 2 = 0$ using the quadratic formula. (Answer: $x = 3 \pm \sqrt{11}$. D = 36+8 = 44; x = (6±√44)/2.)
- Practice: I. $x^2 - x - 12 = 0$ II. $y^2 - 7y + 10 = 0$. Find the relationship between x and y. (Answer: Cannot be determined. x = 4 or −3; y = 2 or 5; pairs give both x>y and x<y.)
- Practice: Determine the nature of the roots of $x^2 - 6x + 9 = 0$ without solving. (Answer: Real and equal roots. D = 36−36 = 0.)
- Practice: Determine the nature of the roots of $2x^2 - 3x + 5 = 0$ without solving. (Answer: No real roots. D = 9−40 = −31 < 0.)

> **Quick revision:** Solve equation I for all values of x, and equation II for all values of y — each quadratic usually gives two roots; Compare every x with every y. If x is bigger in every single combination, the answer is "x > y"; if x is always smaller, it's "x < y"; If some combinations give x bigger and others give x equal, the answer is "x ≥ y" (and symmetrically "x ≤ y") — not "x > y", since equality also occurs; If the combinations genuinely conflict — some give x > y, others x < y (or x = y) — no single relationship covers every case, so the answer is "cannot be determined"; When an equation doesn't factor neatly into integers, use the quadratic formula: for $ax^2 + bx + c = 0$, $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$. The discriminant $D = b^2 - 4ac$ tells you the nature of the roots before you even finish solving: D > 0 gives two distinct real roots, D = 0 gives one repeated real root, D < 0 gives no real roots at all; Shortcut without fully solving: sum of roots = −b/a, and product of roots = c/a. A negative product means the two roots have opposite signs (one positive, one negative) — worth checking early, since a negative root often breaks what would otherwise look like a clean "x > y" or "x < y" pattern; Common mistakes: dropping the ± when taking a square root in the quadratic formula, and sign slips while factoring — always re-expand a factored form mentally to confirm it reproduces the original equation

Simple Equations Solved Questions

The hard part of these questions isn't solving the equation — it's translating the sentence into one. Once the word problem becomes algebra, it's routine. This page covers that translation skill, solving systems with elimination/substitution, consecutive-number problems, and fraction word problems, with several fully worked examples. Assign a variable to the unknown quantity the question actually asks for, and translate each sentence into an equation using that variable (and a second variable if there are two unknowns). Phrases translate directly: "sum of" → +, "difference of" → −, "X more than Y" → Y + X, "X less than Y" → Y − X, "twice/three times a number" →

2x/3x. With two unknowns, you need two independent equations — write both before trying to solve either one. Once solved, substitute your answer back into BOTH original sentences (not just the equation) to confirm it actually satisfies the word problem, not just the algebra. For two unknowns where the equations aren't simply "sum" and "difference" (e.g. mixed quantities at different rates), use elimination — scale one or both equations so a variable's coefficient matches, then subtract to cancel it — or substitution — solve one equation for one variable and plug it into the other. For "consecutive numbers" questions, express them all in terms of one variable with a fixed step: x, x+1, x+2, … for consecutive integers; x, x+2, x+4, … for consecutive even or odd numbers. Their given sum or relationship then becomes one simple equation.

Examples

- The sum of two numbers is 45, and their difference is 11. Find the two numbers. Solution: Let the numbers be x and y: x + y = 45 and x − y = 11. Adding the two equations: 2x = 56, so x = 28. Substituting back: y = 45 − 28 = 17. Answer: 28 and 17
- Five years ago, a father was three times as old as his son. Ten years from now, the father will be twice as old as the son. Find their present ages. Solution: Let the son's present age = s and the father's present age = f. Five years ago: f − 5 = 3(s − 5), so f = 3s − 10. Ten years from now: f + 10 = 2(s + 10), so f = 2s + 10. Setting these equal: 3s − 10 = 2s + 10, so s = 20, and f = 2(20) + 10 = 50. Answer: Father: 50 years, Son: 20 years
- A number is 8 more than twice another number. If the sum of the two numbers is 50, find both numbers. Solution: Let the numbers be x and y, with x = 2y + 8. Substituting into x + y = 50: (2y + 8) + y = 50, so 3y = 42, giving y = 14. Then x = 2(14) + 8 = 36. Answer: 36 and 14
- Three times a number, decreased by 7, equals twice the number increased by 9. Find the number. Solution: Let the number be x: 3x − 7 = 2x + 9. Subtracting 2x from both sides: x − 7 = 9, so x = 16. Answer: 16
- 3 pens and 2 pencils cost ₹78. 5 pens and 3 pencils cost ₹128. Find the cost of one pen and one pencil. Solution: Let a pen cost p and a pencil cost q: 3p + 2q = 78 … (1), and 5p + 3q = 128 … (2). Scale to eliminate q: multiply (1) by 3 and (2) by 2: 9p + 6q = 234, and 10p + 6q = 256. Subtract: p = 256 − 234 = 22. Substitute into (1): 3(22) + 2q = 78 → 66 + 2q = 78 → q = 6. Check in (2): 5(22) + 3(6) = 110 + 18 = 128. Matches. Answer: Pen = ₹22, Pencil = ₹6
- The sum of three consecutive integers is 72. Find the integers. Solution: Let the integers be x, x+1, x+2. Their sum: x + (x+1) + (x+2) = 72 → 3x + 3 = 72 → x = 23. The integers are 23, 24, 25. Answer: 23, 24, 25
- If 3 is added to both the numerator and denominator of a fraction, it becomes 3/4. If 1 is subtracted from both, it becomes 1/2. Find the original fraction. Solution: Let the fraction be x/y. (x+3)/(y+3) = 3/4 → 4x + 12 = 3y + 9 → 4x − 3y = −3 … (1). (x−1)/(y−1) = 1/2 → 2x − 2 = y − 1 → y = 2x − 1 … (2). Substitute (2) into (1): 4x − 3(2x−1) = −3 → 4x − 6x + 3 = −3 → −2x = −6 → x = 3. Then y = 2(3) − 1 = 5. Check: (3+3)/(5+3) = 6/8 = 3/4. (3−1)/(5−1) = 2/4 = 1/2. Both match. Answer: 3/5
- Practice: The sum of two numbers is 60, and their difference is 14. Find the two numbers. (Answer: 37 and 23)
- Practice: A number is 5 more than three times another number. If the sum of the two numbers is 45, find both numbers. (Answer: 35 and 10)
- Practice: Four times a number, decreased by 9, equals the number increased by 21. Find the number. (Answer: 10)
- Practice: 4 notebooks and 3 pens cost ₹84. 3 notebooks and 2 pens cost ₹59. Find the cost of one notebook and one pen. (Answer: Notebook = ₹9, Pen = ₹16. Eliminate using 2×(first) and 3×(second).)
- Practice: The sum of four consecutive integers is 94. Find the largest of the four. (Answer: 25. 4x+6=94 gives x=22; largest is x+3.)
- Practice: The sum of three consecutive even numbers is 72. Find the numbers. (Answer: 22, 24, 26. x+(x+2)+(x+4)=72 gives x=22.)
- Practice: If 2 is added to both the numerator and denominator of a fraction, it becomes 1/2. If 1 is subtracted from both, it becomes 1/3. Find the original fraction. (Answer: 4/10 (= 2/5). Solve y=2x+2 and y=3x-2 together to get x=4, y=10.)

Quick revision: Assign a variable to the unknown quantity the question actually asks for, and translate each sentence into an equation using that variable (and a second variable if there are two unknowns); Phrases translate directly: "sum of" → +, "difference of" → –, "X more than Y" → Y + X, "X less than Y" → Y – X, "twice/three times a number" → 2x/3x; With two unknowns, you need two independent equations — write both before trying to solve either one; Once solved, substitute your answer back into BOTH original sentences (not just the equation) to confirm it actually satisfies the word problem, not just the algebra; For two unknowns where the equations aren't simply "sum" and "difference" (e.g. mixed quantities at different rates), use elimination — scale one or both equations so a variable's coefficient matches, then subtract to cancel it — or substitution — solve one equation for one variable and plug it into the other; For "consecutive numbers" questions, express them all in terms of one variable with a fixed step: x, x+1, x+2, … for consecutive integers; x, x+2, x+4, … for consecutive even or odd numbers. Their given sum or relationship then becomes one simple equation

Algebraic Identities Solved Questions

A handful of fixed identities let you find a^2+b^2 or ab without ever solving for a and b individually — and the same identities turn an awkward multiplication like 97×103 into subtraction. This page covers the square identities, difference of squares, and the sum/difference of cubes, with several fully worked examples. The two core identities: $(a+b)^2 = a^2 + 2ab + b^2$, and $(a–b)^2 = a^2 – 2ab + b^2$. Rearranged, each one connects a^2+b^2 to $(a+b)^2$ or $(a–b)^2$ and ab — given any two of the three, find the third. Difference of squares: $a^2 – b^2 = (a+b)(a–b)$ — this is the shortcut behind multiplying two numbers that are equally far from a round number, like $97×103 = (100–3)(100+3)$. When a question gives a+b (or a–b) and ab, it's testing whether you know $a^2+b^2 = (a+b)^2 – 2ab$ (or $(a–b)^2 + 2ab$) — don't try to find a and b individually first. These identities work in both directions: use them to expand an expression, or to go from a^2+b^2 and a+b (or a–b) back to find ab. Sum and difference of cubes: $a^3+b^3 = (a+b)(a^2–ab+b^2)$, and $a^3–b^3 = (a–b)(a^2+ab+b^2)$ — use these to factor a cube expression, such as $x^3–8 = (x–2)(x^2+2x+4)$. A faster route to a^3+b^3 or $a^3–b^3$ when you already know a+b (or a–b) and ab: $a^3+b^3 = (a+b)^3 – 3ab(a+b)$, and $a^3–b^3 = (a–b)^3 + 3ab(a–b)$ — this avoids separately computing $a^2 \mp ab+b^2$.

Examples

- If a+b = 10 and ab = 21, find $a^2 + b^2$. Solution: $a^2 + b^2 = (a+b)^2 – 2ab. = 10^2 – 2(21) = 100 – 42 = 58$. Answer: 58
- If a–b = 4 and ab = 12, find $a^2 + b^2$. Solution: $a^2 + b^2 = (a–b)^2 + 2ab. = 4^2 + 2(12) = 16 + 24 = 40$. Answer: 40
- Simplify using identities: 97 × 103 Solution: $97 × 103 = (100–3)(100+3) = 100^2 – 3^2. = 10{,}000 – 9 = 9{,}991$. Answer: 9,991
- If a+b = 7 and $a^2 + b^2 = 25$, find ab. Solution: $(a+b)^2 = a^2 + 2ab + b^2$, so $49 = 25 + 2ab$. $2ab = 24$, so ab = 12. Answer: 12
- If a+b = 5 and ab = 6, find $a^3 + b^3$. Solution: $a^3+b^3 = (a+b)^3 – 3ab(a+b). = 5^3 – 3(6)(5) = 125 – 90 = 35$. Answer: 35
- Factor completely: $x^3 – 8$ Solution: Write 8 as 2^3, then apply the difference of cubes: $x^3 – 2^3 = (x – 2)(x^2 + 2x + 4)$. Answer: $(x – 2)(x^2 + 2x + 4)$
- If a–b = 3 and ab = 10, find $a^3 – b^3$. Solution: $a^3–b^3 = (a–b)^3 + 3ab(a–b). = 3^3 + 3(10)(3) = 27 + 90 = 117$. Answer: 117
- Practice: If a+b = 12 and ab = 27, find $a^2 + b^2$. (Answer: 90)
- Practice: Simplify using identities: 95 × 105 (Answer: 9,975)
- Practice: If a–b = 6 and $a^2 + b^2 = 58$, find ab. (Answer: 11)
- Practice: If a+b = 6 and ab = 8, find $a^3 + b^3$. (Answer: 72. $6^3 – 3(8)(6) = 216 – 144$.)
- Practice: Factor completely: $x^3 – 27$ (Answer: $(x – 3)(x^2 + 3x + 9)$. $27 = 3^3$.)
- Practice: If a–b = 2 and ab = 15, find $a^3 – b^3$. (Answer: 98. $2^3 + 3(15)(2) = 8 + 90$.)
- Practice: Simplify using identities: 998 × 1002 (Answer: 999,996. $(1000–2)(1000+2) = 1{,}000{,}000 – 4$.)

Quick revision: The two core identities: $(a+b)^2 = a^2 + 2ab + b^2$, and $(a-b)^2 = a^2 - 2ab + b^2$. Rearranged, each one connects a^2+b^2 to $(a+b)^2$ or $(a-b)^2$ and ab — given any two of the three, find the third; Difference of squares: $a^2 - b^2 = (a+b)(a-b)$ — this is the shortcut behind multiplying two numbers that are equally far from a round number, like $97\times103 = (100-3)(100+3)$; When a question gives $a+b$ (or $a-b$) and ab, it's testing whether you know $a^2+b^2 = (a+b)^2 - 2ab$ (or $(a-b)^2 + 2ab$) — don't try to find a and b individually first; These identities work in both directions: use them to expand an expression, or to go from a^2+b^2 and $a+b$ (or $a-b$) back to find ab; Sum and difference of cubes: $a^3+b^3 = (a+b)(a^2-ab+b^2)$, and $a^3-b^3 = (a-b)(a^2+ab+b^2)$ — use these to factor a cube expression, such as $x^3-8 = (x-2)(x^2+2x+4)$; A faster route to a^3+b^3 or a^3-b^3 when you already know $a+b$ (or $a-b$) and ab: $a^3+b^3 = (a+b)^3 - 3ab(a+b)$, and $a^3-b^3 = (a-b)^3 + 3ab(a-b)$ — this avoids separately computing $a^2 \mp ab+b^2$

Banker's Discount and True Discount

Banker's Discount & True Discount

A bill due in the future is worth less today — True Discount measures that gap properly, while a banker who pays out immediately charges simple interest on the full future amount instead, which is always a little more. This page covers the core formulas, the Banker's Gain shortcuts, and reverse problems recovering the bill amount and rate, with several fully worked examples. Present Worth (PW) is the sum which, invested today at the given rate, would grow into the Amount (A) due at the future date. True Discount (TD) = A − PW. Formula: $PW = 100\times A / (100 + R\times T)$, so TD = A − PW, where R is the annual rate and T is the time in years. Banker's Discount (BD) is simpler but less accurate — it's just the simple interest on the full future Amount for the unexpired time: $BD = A\times R\times T / 100$. BD is always ≥ TD. Banker's Gain (BG) = BD − TD — the extra the banker effectively earns by charging interest on the full future amount instead of the smaller present worth. Useful shortcuts once you already have some of these values: Banker's Gain = $TD^2 / PW = (BD \times TD) / A$. These let you find BG directly without separately working out R and T. Reverse problems that give you BD and TD (or BG) directly can be solved step by step: first find BG = BD − TD, then $PW = TD^2/BG$, then A = PW + TD, and finally use $TD = A\times RT/(100+RT)$ to recover the rate (for a given time) or the time (for a given rate).

Examples

- Find the present worth (PW) and the true discount (TD) on ₹1,200 due 2 years hence at 10% per annum simple interest. Solution: $PW = 100 \times 1{,}200 / (100 + 10\times2) = 120{,}000 / 120 = ₹1{,}000$. TD = A − PW = 1,200 − 1,000 = ₹200. Answer: PW = ₹1,000, TD = ₹200
- Find the banker's discount (BD) on ₹1,200 due 2 years hence at 10% per annum. Solution: $BD = A\times R\times T / 100 = 1{,}200 \times 10 \times 2 / 100 = ₹240$. Answer: ₹240
- For the same bill, BD = ₹240 and TD = ₹200. Find the banker's gain (BG). Solution: BG = BD − TD = 240 − 200 = ₹40. Answer: ₹40
- The banker's discount on a bill due 2 years hence at 10% per annum is ₹360. Find the true discount. Solution: From BD, find the Amount first: $A = BD \times 100 / (R\times T) = 360 \times 100 / 20 = ₹1{,}800$. $TD = A \times R\times T / (100 + R\times T) = 1{,}800 \times 20 / 120 = ₹300$. Answer: ₹300
- If the true discount on a bill is ₹40 and the present worth is ₹400, find the banker's gain using the shortcut formula. Solution: $BG = TD^2 / PW. = 40^2 / 400 = 1{,}600 / 400 = ₹4$. Answer: ₹4
- On a bill of ₹2,200, the true discount is ₹200 and the banker's discount is ₹220. Verify the banker's gain using both BG = BD − TD and $BG = (BD \times TD)/A$. Solution: Directly: BG = BD − TD = 220 − 200 = ₹20. Shortcut: $BG = (BD \times TD)/A = (220 \times 200)/2{,}200 = 44{,}000/2{,}200 = ₹20$. Both methods agree. Answer: ₹20
- The banker's discount and true discount on a bill due in 1 year are ₹120 and ₹100 respectively. Find the amount of the bill and the rate of interest. Solution: BG = BD − TD = 120 − 100 = 20. Then $PW = TD^2/BG = 100^2/20 = 10{,}000/20 = ₹500$. Amount = PW + TD = 500 + 100 = ₹600. Using $TD = A\times RT/(100+RT)$ with T = 1: $100 = 600R/(100+R) \rightarrow 10{,}000 + 100R = 600R \rightarrow R = 10{,}000/500 = 20\%$. Answer: Amount = ₹600, Rate = 20% per annum
- Practice: Find the present worth and the true discount on ₹2,400 due 2 years hence at 10% per annum simple interest. (Answer: PW = ₹2,000, TD = ₹400)
- Practice: Find the banker's discount on ₹2,400 due 2 years hence at 10% per annum. (Answer: ₹480)

- Practice: The banker’s discount on a bill due 2 years hence at 10% per annum is ₹480. Find the true discount. (Answer: ₹400)
- Practice: If the true discount on a bill is ₹60 and the present worth is ₹900, find the banker's gain using the shortcut formula. (Answer: ₹4. BG = TD^2/PW = 3,600/900.)
- Practice: On a bill of ₹1,320, the true discount is ₹120 and the banker's discount is ₹132. Verify the banker's gain using both formulas. (Answer: ₹12 by both methods: 132−120, and (132×120)/1,320.)
- Practice: The banker's discount and true discount on a bill due in 1 year are ₹216 and ₹200 respectively. Find the amount of the bill and the rate of interest. (Answer: Amount = ₹2,700, Rate = 8%. BG=16, PW=TD^2/BG=2,500, A=PW+TD=2,700; then solve 200=2,700R/(100+R).)
- Practice: Find the present worth and true discount on ₹3,600 due 2 years hence at 10% per annum simple interest. (Answer: PW = ₹3,000, TD = ₹600.)

> **Quick revision:** Present Worth (PW) is the sum which, invested today at the given rate, would grow into the Amount (A) due at the future date. True Discount (TD) = A − PW; Formula: PW = 100×A / (100 + R×T), so TD = A − PW, where R is the annual rate and T is the time in years; Banker’s Discount (BD) is simpler but less accurate — it’s just the simple interest on the full future Amount for the unexpired time: BD = A×R×T / 100. BD is always ≥ TD; Banker’s Gain (BG) = BD − TD — the extra the banker effectively earns by charging interest on the full future amount instead of the smaller present worth; Useful shortcuts once you already have some of these values: Banker's Gain = TD^2 / PW = (BD × TD) / A. These let you find BG directly without separately working out R and T; Reverse problems that give you BD and TD (or BG) directly can be solved step by step: first find BG = BD − TD, then PW = TD^2/BG, then A = PW + TD, and finally use TD = A×RT/(100+RT) to recover the rate (for a given time) or the time (for a given rate)

Boat and Stream

Boat and Stream: downstream and upstream speed

If a boat's speed in still water is B and the stream's speed is S: downstream speed (with the current) = B + S, and upstream speed (against the current) = B − S. Given the downstream speed (D) and upstream speed (U), the boat's and stream's own speeds can be recovered by reversing the formula: boat speed = (D + U) / 2, stream speed = (D − U) / 2. Once a downstream or upstream speed is known, it becomes an ordinary Time, Speed and Distance problem - apply Time = Distance / Speed as usual. For a round trip covering the same distance downstream and back upstream, use the two-unequal-speeds average-speed shortcut: average speed = 2 × downstream speed × upstream speed / (downstream speed + upstream speed).

A useful ratio shortcut: if a boat takes a certain time ratio to cover the same distance upstream versus downstream, the speed ratio is the exact inverse of the time ratio (since speed and time are inversely proportional for a fixed distance). So if the upstream time is twice the downstream time, the downstream speed is twice the upstream speed - and working backward through B=(D+U)/2 and S=(D-U)/2 shows the boat's still-water speed is then exactly 3 times the stream's speed. This lets many "ratio of times" questions be solved without ever finding the actual distance.

A constraint worth remembering: the stream's speed must always be strictly less than the boat's own speed (S < B). If the stream were as fast as or faster than the boat, the upstream speed B − S would be zero or negative, meaning the boat could never make progress against the current at all - a boat speed answer that comes out smaller than the stream speed signals an arithmetic error, not a valid scenario.

For round-trip problems that give a one-way distance directly (rather than just a total time), simply add the downstream and upstream travel times separately using Time = Distance/Speed for each leg - there is no shortcut formula needed when the distance is already known; the average-speed shortcut above is only for when distance is unknown but total time is.

Examples

- A boat's speed in still water is 10 km/hr and the stream's speed is 4 km/hr: downstream speed = 10 + 4 = 14 km/hr, upstream speed = 10 − 4 = 6 km/hr.

- A boat rows 24 km downstream in 2 hours (downstream speed = 24/2 = 12 km/hr); if its still-water speed is 9 km/hr, the stream's speed = 12 − 9 = 3 km/hr.
- A boat's downstream speed is 18 km/hr and upstream speed is 12 km/hr: still-water speed = (18+12)/2 = 15 km/hr, stream speed = (18−12)/2 = 3 km/hr, and the average round-trip speed = 2×18×12/(18+12) = 14.4 km/hr.
- If a boat takes twice as long to travel a certain distance upstream as it does downstream, the downstream speed must be twice the upstream speed (D = 2U); working through B=(D+U)/2 and S=(D-U)/2 shows the boat's still-water speed is exactly 3 times the stream's speed.
- A boat's speed in still water is 9 km/hr and the stream's speed is 3 km/hr. For a 24 km trip each way: downstream speed = 12 km/hr takes 24/12 = 2 hours, upstream speed = 6 km/hr takes 24/6 = 4 hours, so the total round-trip time is 2 + 4 = 6 hours.
- A stream speed can never equal or exceed the boat's own speed: if a calculation gives an upstream speed of 0 or a negative number, the boat and stream speeds have been mixed up somewhere, since a real boat can always make at least some progress against a real stream.
- A boat's speed in still water is 12 km/hr, and the stream's speed is 3 km/hr: its upstream speed is 12 − 3 = 9 km/hr - note this is still positive, confirming 3 < 12 as required.

> **Quick revision:** Downstream = boat + stream; Upstream = boat − stream. Boat speed = (D+U)/2; Stream speed = (D−U)/2. Round-trip average speed = 2DU/(D+U). Time ratio upstream:downstream inverts to give the speed ratio. Stream speed must always be less than boat speed (S < B).

Boat and Stream Solved Questions

Boat and Stream questions are Time, Speed and Distance questions wearing a different costume — convert the boat's and stream's speeds into a downstream or upstream speed first, then it's the same formulas you already know. This page covers that conversion, round trips given total time, and the time-ratio shortcut, with several fully worked examples. If a boat's speed in still water is B and the stream's speed is S: downstream speed (with the current) = B + S. Upstream speed (against the current) = B − S. Given the downstream speed (D) and upstream speed (U), you can recover the boat's and stream's own speeds by reversing the formula: boat speed = (D + U) / 2, stream speed = (D − U) / 2. Once you have a downstream or upstream speed, it's an ordinary Time, Speed and Distance problem — apply Time = Distance / Speed as usual. For a round trip covering the same distance downstream and back upstream, use the same two-unequal-speeds average-speed shortcut from Time, Speed and Distance: average speed = 2 × downstream speed × upstream speed / (downstream speed + upstream speed). If a question gives the TOTAL time for a round trip instead of one-way data, set up one equation: total time = distance/downstream speed + distance/upstream speed, and solve it for the unknown (usually the distance). If the time taken upstream is some multiple n of the time taken downstream for the same distance, then downstream speed = n × upstream speed (since time and speed are inversely related for a fixed distance) — this gives boat speed : stream speed = (n + 1) : (n − 1). Read carefully whether a stated "speed of the boat" means its still-water speed or an already-combined downstream/upstream speed — the wording decides which formula to apply first.

Examples

- A boat's speed in still water is 10 km/hr, and the speed of the stream is 4 km/hr. Find the boat's downstream and upstream speeds. Solution: Downstream speed = boat speed + stream speed = 10 + 4 = 14 km/hr. Upstream speed = boat speed − stream speed = 10 − 4 = 6 km/hr. Answer: Downstream = 14 km/hr, Upstream = 6 km/hr
- A boat can row 24 km downstream in 2 hours. If the boat's speed in still water is 9 km/hr, find the speed of the stream. Solution: Downstream speed = Distance / Time = 24 / 2 = 12 km/hr. Stream speed = downstream speed − boat speed = 12 − 9 = 3 km/hr. Answer: 3 km/hr
- A man can row 15 km upstream in 3 hours, and his speed in still water is 7 km/hr. Find the speed of the stream and the time he would take to cover the same 15 km downstream. Solution: Upstream speed = Distance / Time = 15 / 3 = 5 km/hr. Stream speed = boat speed − upstream speed = 7 − 5 = 2 km/hr. Downstream speed = boat speed + stream speed = 7 + 2 = 9 km/hr. Time downstream = Distance / Speed = 15 / 9 = 5/3 hours = 1 hour 40 minutes. Answer: Stream speed = 2 km/hr, Time downstream = 1 hour 40 minutes

- A boat's downstream speed is 18 km/hr and its upstream speed is 12 km/hr. Find the boat's speed in still water, the speed of the stream, and the boat's average speed for a round trip covering the same distance each way. Solution: Boat speed = (downstream + upstream) / 2 = (18 + 12) / 2 = 15 km/hr. Stream speed = (downstream − upstream) / 2 = (18 − 12) / 2 = 3 km/hr. Average round-trip speed = 2 × 18 × 12 / (18 + 12) = 432 / 30 = 14.4 km/hr. Answer: Boat speed = 15 km/hr, Stream speed = 3 km/hr, Average round-trip speed = 14.4 km/hr
- A man rows downstream at 12 km/hr and upstream at 8 km/hr. He rows to a certain place and back, taking a total of 5 hours. Find the distance to that place. Solution: Let the distance be d. Total time = d/12 + d/8 = 5. d(1/12 + 1/8) = d(2/24 + 3/24) = d(5/24) = 5. d = 5 × 24/5 = 24 km. Answer: 24 km
- A boat takes twice as long to go a certain distance upstream as it does to cover the same distance downstream. If the boat's speed in still water is 15 km/hr, find the speed of the stream. Solution: Time upstream = 2 × time downstream for the same distance, so downstream speed = 2 × upstream speed, i.e. n = 2. Boat speed : stream speed = (n+1) : (n−1) = 3 : 1. Since boat speed = 15, stream speed = 15/3 = 5 km/hr. Check: downstream = 15+5 = 20, upstream = 15−5 = 10; downstream/upstream = 2, matching the time ratio. Answer: 5 km/hr
- A boat covers 36 km upstream in 3 hours, and the same 36 km downstream in 2 hours. Find the boat's speed in still water and the speed of the stream. Solution: Upstream speed = 36/3 = 12 km/hr. Downstream speed = 36/2 = 18 km/hr. Boat speed = (18 + 12)/2 = 15 km/hr. Stream speed = (18 − 12)/2 = 3 km/hr. Answer: Boat speed = 15 km/hr, Stream speed = 3 km/hr
- Practice: A boat's speed in still water is 12 km/hr, and the speed of the stream is 3 km/hr. Find the boat's downstream and upstream speeds. (Answer: Downstream = 15 km/hr, Upstream = 9 km/hr)
- Practice: A boat can row 32 km downstream in 4 hours. If the boat's speed in still water is 6 km/hr, find the speed of the stream. (Answer: 2 km/hr)
- Practice: A boat's downstream speed is 20 km/hr and its upstream speed is 10 km/hr. Find the boat's average speed for a round trip covering the same distance each way. (Answer: 40/3 ≈ 13.33 km/hr)
- Practice: A man rows downstream at 15 km/hr and upstream at 9 km/hr. He rows to a place and back in a total of 4 hours. Find the distance to the place. (Answer: 22.5 km. d(1/15+1/9) = d(8/45) = 4, so d = 4×45/8.)
- Practice: A boat's speed in still water is 20 km/hr. It takes three times as long to row upstream as downstream for the same distance. Find the speed of the stream. (Answer: 10 km/hr. Boat:stream = (3+1):(3−1) = 4:2 = 2:1, so stream = 20/2.)
- Practice: A boat covers 24 km upstream in 4 hours and the same distance downstream in 2 hours. Find the boat's speed in still water and the stream's speed. (Answer: Boat = 9 km/hr, Stream = 3 km/hr. Upstream = 6 km/hr, downstream = 12 km/hr.)
- Practice: Using the boat and stream from the previous question (downstream 12 km/hr, upstream 6 km/hr), find its average speed for a round trip over equal distances. (Answer: 8 km/hr. 2×12×6/(12+6) = 144/18.)

> **Quick revision:** If a boat's speed in still water is B and the stream's speed is S: downstream speed (with the current) = B + S. Upstream speed (against the current) = B − S; Given the downstream speed (D) and upstream speed (U), you can recover the boat's and stream's own speeds by reversing the formula: boat speed = (D + U) / 2, stream speed = (D − U) / 2; Once you have a downstream or upstream speed, it's an ordinary Time, Speed and Distance problem — apply Time = Distance / Speed as usual; For a round trip covering the same distance downstream and back upstream, use the same two-unequal-speeds average-speed shortcut from Time, Speed and Distance: average speed = 2 × downstream speed × upstream speed / (downstream speed + upstream speed); If a question gives the TOTAL time for a round trip instead of one-way data, set up one equation: total time = distance/downstream speed + distance/upstream speed, and solve it for the unknown (usually the distance); If the time taken upstream is some multiple n of the time taken downstream for the same distance, then downstream speed = n × upstream speed (since time and speed are inversely related for a fixed distance) — this gives boat speed : stream speed = (n + 1) : (n − 1); Read carefully whether a stated "speed of the boat" means its still-water speed or an already-combined downstream/upstream speed — the wording decides which formula to apply first

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Chain Rule

Chain Rule (Compound Proportion)

Chain Rule questions involve three or more related quantities at once (workers, days, hours; or men and days) — the method is to set up a direct or inverse proportion for each pair and chain them together, not to guess which way to multiply. This page covers that method, ration/food-duration problems, and cases where the total work itself also changes, with several fully worked examples. Identify whether each pair of quantities is DIRECTLY proportional (both increase together, like cost and quantity) or INVERSELY proportional (one increases as the other decreases, like workers and days for a fixed job). More workers → fewer days needed (inverse). More hours per day → fewer days needed (inverse). More work to do → more days needed (direct). Set up the unknown as: new value = old value × (ratio that makes sense for each relationship) — for an inverse relationship, the ratio is old/new; for a direct relationship, it's new/old. Chain every relevant ratio together in one single equation rather than solving pair by pair — this avoids rounding errors and mixed-up intermediate steps. Food/ration-duration problems are an inverse-proportion chain rule in disguise: more people sharing a fixed stock of food means it lasts fewer days. If the headcount changes partway through, split into stages — work out how much stock is used before the change, how much is left, and divide that remainder by the new headcount. If the total amount of WORK itself also changes (not just workers/days/hours) — e.g. a longer wall, or a bigger task — add one more direct-proportion factor to the chain: (new work amount / old work amount), since more work needs more days, all else equal.

Examples

- If 15 workers can build a wall in 12 days working 8 hours a day, how many days will 18 workers take to build the same wall working 10 hours a day? Solution: More workers → fewer days (inverse): multiply by 15/18. More hours per day → fewer days (inverse): multiply by 8/10. Days = 12 × (15/18) × (8/10) = 12 × (5/6) × (4/5) = 8 days. Answer: 8 days
- The cost of 8 pens equals the cost of 6 pencils. If the cost of 10 pencils is ₹150, find the cost of 20 pens. Solution: Cost of 1 pencil = 150 / 10 = ₹15, so the cost of 6 pencils = 6 × 15 = ₹90. The cost of 8 pens equals this: 8 pens = ₹90, so 1 pen = 90/8 = ₹11.25. Cost of 20 pens = 20 × 11.25 = ₹225. Answer: ₹225
- 36 men can complete a piece of work in 18 days. In how many days will 27 men complete the same work? Solution: More men → fewer days (inverse proportion): days = 36 × 18 / 27. = 648 / 27 = 24 days. Answer: 24 days
- A car covers a certain distance in 5 hours at a speed of 60 km/hr. If the speed is increased to 75 km/hr, find the time taken to cover the same distance. Solution: The distance stays fixed, and speed and time are inversely proportional: time = (5 × 60) / 75. = 300 / 75 = 4 hours. Answer: 4 hours
- A garrison of 500 men has food that will last 30 days. After 10 days, 100 more men join. For how many more days will the remaining food last? Solution: Total food stock = 500 × 30 = 15,000 "man-days". Used in the first 10 days = 500 × 10 = 5,000 man-days. Remaining = 15,000 − 5,000 = 10,000 man-days. New headcount = 500 + 100 = 600. Remaining days = 10,000 / 600 ≈ 16.67 days. Answer: ≈ 16.67 days (16 days and 16 hours)
- If 20 men can build a 50 m long wall in 12 days, how many days will 15 men take to build a 75 m long wall (at the same rate of work)? Solution: Chain the ratios: fewer men → more days (inverse, multiply by 20/15); more wall to build → more days (direct, multiply by 75/50). Days = 12 × (20/15) × (75/50) = 12 × (4/3) × (3/2) = 12 × 2 = 24 days. Answer: 24 days
- If 6 men can earn ₹9,000 in 5 days, how much will 8 men earn in 10 days, at the same daily wage rate per man? Solution: Earnings scale up directly with both the number of men and the number of days. New earnings = 9,000 × (8/6) × (10/5) = 9,000 × (4/3) × 2 = 9,000 × 8/3 = ₹24,000. Answer: ₹24,000
- Practice: 20 workers can build a wall in 15 days working 6 hours a day. How many days will 15 workers take to build the same wall working 8 hours a day? (Answer: 15 days)
- Practice: The cost of 12 notebooks is ₹180. Find the cost of 20 notebooks. (Answer: ₹300)
- Practice: 24 men can complete a job in 20 days. In how many days will 30 men complete the same job? (Answer: 16 days)
- Practice: A ship has provisions for 400 passengers for 60 days. After 15 days, 100 passengers leave the ship. For how many more days will the remaining food last? (Answer: 60 days. Stock = 24,000 man-days; used = 6,000; remaining = 18,000 ÷ 300 passengers.)

- Practice: If 25 men can build a 40 m long wall in 16 days, how many days will 20 men take to build a 60 m long wall? (Answer: 30 days. 16 × (25/20) × (60/40) = 16 × 1.25 × 1.5.)
- Practice: If 5 men can earn ₹6,000 in 8 days, how much will 10 men earn in 6 days, at the same daily wage rate? (Answer: ₹9,000. 6,000 × (10/5) × (6/8).)
- Practice: 15 workers can complete a task in 8 days working 6 hours a day. How many days will 12 workers take, working 10 hours a day? (Answer: 6 days. 8 × (15/12) × (6/10).)

> **Quick revision:** Identify whether each pair of quantities is DIRECTLY proportional (both increase together, like cost and quantity) or INVERSELY proportional (one increases as the other decreases, like workers and days for a fixed job); More workers → fewer days needed (inverse). More hours per day → fewer days needed (inverse). More work to do → more days needed (direct); Set up the unknown as: new value = old value × (ratio that makes sense for each relationship) — for an inverse relationship, the ratio is old/new; for a direct relationship, it's new/old; Chain every relevant ratio together in one single equation rather than solving pair by pair — this avoids rounding errors and mixed-up intermediate steps; Food/ration-duration problems are an inverse-proportion chain rule in disguise: more people sharing a fixed stock of food means it lasts fewer days. If the headcount changes partway through, split into stages — work out how much stock is used before the change, how much is left, and divide that remainder by the new headcount; If the total amount of WORK itself also changes (not just workers/days/hours) — e.g. a longer wall, or a bigger task — add one more direct-proportion factor to the chain: (new work amount / old work amount), since more work needs more days, all else equal

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Geometry: Circles

Geometry: Circles — Concepts and Applications

A perpendicular from a circle's centre to a chord bisects that chord, and equal chords in the same circle have equal perpendicular distances from the centre. The angle at the centre is twice an inscribed angle subtending the same arc, and inscribed angles standing on the same chord from the same segment are equal. Opposite angles of a cyclic quadrilateral sum to 180°. A radius to a point of tangency is perpendicular to the tangent, and tangent segments from one external point to the same circle have equal lengths. For circles with centre distance d and radii R and r, a direct common tangent segment has length $\sqrt{(d^2 - (R - r)^2)}$, while a transverse common tangent segment has length $\sqrt{(d^2 - (R + r)^2)}$ when the stated separate-circle configuration permits it.

The two common-tangent formulas are easy to mix up, so remember the geometric reason behind each: a DIRECT tangent touches both circles on the same side (like two circles sitting side by side with a tangent along the top), so the relevant right-triangle leg is the DIFFERENCE of radii (R − r). A TRANSVERSE tangent crosses between the circles (touching one on top and the other on the bottom), so the relevant leg is the SUM of radii (R + r) - and this only exists when the circles don't overlap.

The two-tangent-equal-length fact (tangent segments from one external point are equal) is often the key unlocking step in circle problems involving an external point touching a circle twice - look for it whenever a single external point has two tangent lines drawn to the same circle.

Examples

- A circle has radius 10 cm. A chord is at perpendicular distance 6 cm from its centre. Find the full chord length in cm. Worked solution: The perpendicular bisects the chord. Half-chord^2 = $10^2 - 6^2$ = 64, so half-chord = 8 cm. Doubling gives 16 cm.
- A minor arc AB subtends 64° at the centre. Point C lies on the major arc AB. Find angle ACB in degrees. Worked solution: Angle ACB intercepts the minor arc AB. An inscribed angle is half the central angle on that same arc, so ACB = 64/2 = 32°.
- Two disjoint circles have radii 16 and 6 cm and centre distance 26 cm. Find the length in cm between contact points on a direct common tangent. Worked solution: A direct tangent gives a right triangle with hypotenuse 26 and perpendicular leg R − r = 10. Tangent length = $\sqrt{(26^2 - 10^2)}$ = 24 cm.

- Two circles of radii 8 cm and 4 cm have their centres 15 cm apart, and they don't overlap. Find the length of a transverse common tangent. Worked solution: A transverse tangent uses the SUM of radii as the relevant leg: length = $\sqrt{(15^2 - (8+4)^2)} = \sqrt{(225-144)} = \sqrt{81} = 9$ cm.
- From an external point P, a tangent is drawn to a circle of radius 5 cm, where P is 13 cm from the centre. Find the tangent length. Worked solution: The radius to the point of tangency is perpendicular to the tangent, forming a right triangle with hypotenuse 13 and one leg 5. Tangent length = $\sqrt{(13^2-5^2)} = \sqrt{(169-25)} = \sqrt{144} = 12$ cm - the classic 5-12-13 triple.
- ABCD is a cyclic quadrilateral with angle A = 70°. Find angle C. Worked solution: Opposite angles of a cyclic quadrilateral sum to 180°, so angle C = 180° − 70° = 110°.

> **Quick revision:** Perpendicular from centre bisects a chord. Central angle = 2 × inscribed angle on the same arc. Opposite angles of a cyclic quadrilateral sum to 180°. Tangent ⊥ radius at the point of contact; two tangents from one external point are equal. Direct common tangent uses (R−r); transverse common tangent uses (R+r), only when circles are separate.

Geometry: Lines and Triangles

Geometry: Lines and Triangles — Concepts and Applications

Adjacent angles on a straight line sum to 180°, vertically opposite angles are equal, and parallel lines cut by a transversal produce equal corresponding and alternate interior angles. A triangle's interior angles sum to 180°, and an exterior angle equals the sum of the two remote interior angles. Congruent triangles have equal corresponding sides and angles, while similar triangles have a common side scale and the square of that scale for areas. A right triangle satisfies $a^2 + b^2 = c^2$ with c the hypotenuse, and its hypotenuse midpoint is equidistant from the three vertices. An altitude is perpendicular to the opposite side, a median joins a vertex to the opposite midpoint, and an internal angle bisector divides the opposite side in the ratio of the adjacent sides. The centroid divides each median 2:1 from the vertex, the incentre is the angle-bisector intersection, the circumcentre is the perpendicular-bisector intersection, and the orthocentre is the altitude intersection.

The Pythagorean relation also works in reverse (the converse): if a triangle's three sides satisfy $a^2 + b^2 = c^2$, the triangle MUST be right-angled at the vertex opposite the longest side c - a quick way to test whether a given set of three side lengths forms a right triangle without any angle information.

The angle bisector theorem gives a concrete ratio: if the bisector from vertex A meets the opposite side BC at point D, then BD:DC = AB:AC - the point divides the opposite side in exactly the same ratio as the two adjacent sides, which turns many "find this length" problems into a straightforward ratio split.

Examples

- Rays OA and OB form a straight line. Ray OC lies on one side of it, with angle AOC = 43°. Find angle COB in degrees. Worked solution: The two adjacent angles form a straight angle of 180°. Therefore angle COB = 180 − 43 = 137°.
- Triangles ABC and DEF are similar with corresponding sides in ratio ABC:DEF = 2:3. If BC = 7 cm and corresponds to EF, find EF in cm. Worked solution: The side multiplier from ABC to DEF is 3/2. Hence EF = 7 × 3/2 = 10.5 cm.
- I is the incentre of triangle ABC and angle A = 42°. Find the smaller angle BIC in degrees. Worked solution: In triangle BIC, the other two angles are B/2 and C/2. Their sum is (180 − 42)/2, so BIC = 180 − (180 − 42)/2 = 111°.
- A triangle has sides 9 cm, 12 cm, and 15 cm. Determine whether it is right-angled. Worked solution: Check if the sides satisfy the Pythagorean relation with the longest side as hypotenuse: $9^2 + 12^2 = 81 + 144 = 225 = 15^2$. Since this holds, the triangle IS right-angled, with the right angle opposite the 15 cm side.
- A median of a triangle has length 12 cm. Find the distance from the centroid to the vertex, and from the centroid to the midpoint of the opposite side. Worked solution: The centroid divides each median 2:1 from the vertex, so vertex-to-centroid = (2/3)×12 = 8 cm, and centroid-to-midpoint = (1/3)×12 = 4 cm.
- In triangle ABC, AB = 6 cm and AC = 9 cm. The internal bisector from A meets BC at D, where BC = 10 cm. Find BD and DC. Worked solution: By the angle bisector theorem, BD:DC = AB:AC = 6:9 = 2:3. Splitting BC = 10 cm in this ratio: BD = (2/5)×10 = 4 cm, DC = (3/5)×10 = 6 cm.

- A triangle's two remote interior angles are 50° and 70°. Find the exterior angle at the third vertex. Worked solution: An exterior angle equals the sum of the two remote interior angles: 50° + 70° = 120°.

> **Quick revision:** Straight line angles sum to 180°; vertically opposite angles are equal; triangle angles sum to 180°; exterior angle = sum of remote interior angles. Similar triangles: sides scale linearly, areas scale by the square. Pythagorean converse: $a^2+b^2=c^2$ means the triangle is right-angled opposite c. Angle bisector theorem: BD:DC = AB:AC. Centroid divides each median 2:1 from the vertex.

Geometry: Lines, Angles & Triangles

This slice of Geometry runs on a small set of fixed facts — the angle sum of a triangle, how an exterior angle relates to the two interior angles across from it, and how a transversal behaves when it cuts two parallel lines. This page covers all of these, plus the triangle inequality and isosceles/equilateral angle properties, with several fully worked examples. The angles of any triangle always add up to 180°. Given any two angles, subtract their sum from 180° to find the third. When angles are given as a ratio (like 2:3:4), assign each part a variable (2x, 3x, 4x), set their sum equal to 180°, solve for x, then multiply back to get each actual angle. The exterior angle of a triangle (formed by extending one side) equals the SUM of the two interior angles that are not adjacent to it — this is faster than working out the third interior angle first and subtracting from 180°. An exterior angle and its adjacent interior angle always add up to 180° (they form a straight line) — useful for converting between the two when a question mixes them. Triangle inequality: the sum of any two sides of a triangle must be greater than the third side. Use this to check whether three given lengths can even form a triangle before doing any angle work. In an isosceles triangle, the two angles opposite the two equal sides are themselves equal; in an equilateral triangle, all three sides are equal and all three angles are 60°. When a transversal cuts two parallel lines: corresponding angles are equal, alternate angles are equal, and co-interior (allied) angles add up to 180°. These three facts are usually what let you find an unknown angle in a parallel-lines figure.

Examples

- In triangle ABC, angle A = 50° and angle B = 70°. Find angle C. Solution: The three angles of a triangle sum to 180°: angle C = 180° − 50° − 70° = 60°. Answer: 60°
- Two angles of a triangle are in the ratio 2:3, and the third angle is 100°. Find the two angles. Solution: Let the two angles be 2x and 3x: 2x + 3x + 100° = 180°, so 5x = 80°, giving x = 16°. The two angles are 2(16°) = 32° and 3(16°) = 48°. Answer: 32° and 48°
- The exterior angle of a triangle is 110°, and one of the interior opposite angles is 40°. Find the other interior opposite angle. Solution: The exterior angle equals the sum of the two non-adjacent interior angles: 110° = 40° + (other angle), so the other angle = 70°. Answer: 70°
- In a triangle, the three angles are in the ratio 1:2:3. Find the largest angle. Solution: Let the angles be x, 2x and 3x: x + 2x + 3x = 180°, so 6x = 180°, giving x = 30°. The largest angle is 3(30°) = 90°. Answer: 90°
- Can a triangle have sides of length 4 cm, 5 cm and 10 cm? Solution: Check the triangle inequality: the sum of the two smaller sides must exceed the largest side. 4 + 5 = 9, which is less than 10. Since the inequality fails, no such triangle can exist. Answer: No — 4 + 5 = 9 < 10, so the triangle inequality fails
- In an isosceles triangle, the vertex angle is 40°. Find each of the two base angles. Solution: The two base angles are equal. Their combined measure = 180° − 40° = 140°. Each base angle = 140° / 2 = 70°. Answer: 70° each
- Two parallel lines are cut by a transversal. One of the angles formed is 70°. Find (a) its corresponding angle on the other parallel line, and (b) the co-interior angle adjacent to that corresponding angle. Solution: (a) Corresponding angles are equal, so the corresponding angle is also 70°. (b) Co-interior angles add up to 180°, so the adjacent co-interior angle = 180° − 70° = 110°. Answer: (a) 70° (b) 110°
- Practice: In triangle PQR, angle P = 65° and angle Q = 55°. Find angle R. (Answer: 60°)
- Practice: Two angles of a triangle are in the ratio 3:4, and the third angle is 40°. Find the two angles. (Answer: 60° and 80°)
- Practice: The exterior angle of a triangle is 125°, and one of the interior opposite angles is 55°. Find the other interior opposite angle. (Answer: 70°)

- Practice: Can a triangle have sides of length 6 cm, 7 cm and 14 cm? (Answer: No. 6 + 7 = 13 < 14, so the triangle inequality fails.)
- Practice: In an isosceles triangle, the vertex angle is 50°. Find each base angle. (Answer: 65° each. (180° − 50°) / 2.)
- Practice: Two parallel lines are cut by a transversal. One angle formed is 110°. Find its alternate angle and its co-interior angle. (Answer: Alternate = 110° (equal); co-interior = 70° (180° − 110°).)
- Practice: The angles of a triangle are in the ratio 2:3:5. Find all three angles. (Answer: 36°, 54° and 90°. 10x = 180° gives x = 18°.)

> **Quick revision:** The angles of any triangle always add up to 180°. Given any two angles, subtract their sum from 180° to find the third; When angles are given as a ratio (like 2:3:4), assign each part a variable (2x, 3x, 4x), set their sum equal to 180°, solve for x, then multiply back to get each actual angle; The exterior angle of a triangle (formed by extending one side) equals the SUM of the two interior angles that are not adjacent to it — this is faster than working out the third interior angle first and subtracting from 180°; An exterior angle and its adjacent interior angle always add up to 180° (they form a straight line) — useful for converting between the two when a question mixes them; Triangle inequality: the sum of any two sides of a triangle must be greater than the third side. Use this to check whether three given lengths can even form a triangle before doing any angle work; In an isosceles triangle, the two angles opposite the two equal sides are themselves equal; in an equilateral triangle, all three sides are equal and all three angles are 60°; When a transversal cuts two parallel lines: corresponding angles are equal, alternate angles are equal, and co-interior (allied) angles add up to 180°. These three facts are usually what let you find an unknown angle in a parallel-lines figure

Free practice questions: parikshasaathi.com/q/geometry-lines-triangles/

Heights and Distances

Heights and Distances — Concepts and Applications

An angle of elevation is measured upwards from the observer's horizontal line of sight, and an angle of depression is measured downwards from it. With level ground and a vertical object, tanθ equals the vertical rise from the observer's eye to the target divided by horizontal distance. Add the observer's eye height when the requested height is measured from the ground. Standard values tan30° = 1/√3, tan45° = 1 and tan60° = √3 connect the usual right-triangle configurations. For two observations, write one equation per position and use the stated distance between positions with the correct same-side or opposite-side arrangement. A shadow and vertical object form another right triangle, while an inclined ladder uses sine when its length is the hypotenuse.

The classic "two-position" problem - where the angle of elevation changes as the observer walks toward or away from the object - needs one tan equation for each position, both referencing the SAME unknown height, with known horizontal distances that differ by the stated walking distance. Solve the two equations simultaneously by expressing both horizontal distances in terms of the height, then using their difference to pin down the height directly.

Examples

- On level ground, an observer at ground level stands 12 m from the foot of a vertical tower. The angle of elevation of its top is 45°. Find the tower height in metres. Worked solution: Height/horizontal distance = tan45° = 1. Therefore height = 12 × 1 = 12 m.
- From the top of a vertical 21 m tower, the angle of depression of a point on level ground is 45°. Find the horizontal distance from that point to the tower's foot, in metres. Worked solution: The ground point sees the tower at elevation 45° because the horizontal lines are parallel. Thus 21/distance = tan45° = 1, giving distance 21 m.
- A straight ladder of length $4\sqrt{2}$ m rests against a vertical wall on level ground, making 45° with the ground. Find the height reached on the wall in metres. Worked solution: The ladder is the hypotenuse. Vertical height = $4\sqrt{2} \times \sin45° = 4\sqrt{2}/\sqrt{2} = 4$ m.
- An observer walks 40 m towards a tower, and the angle of elevation of its top changes from 30° to 60°. Find the tower's height. Worked solution: Let height = h. At the farther position, $h = d_1 \times \tan30°$ so $d_1 = h\sqrt{3}$. At the nearer position, $h = d_2 \times \tan60°$ so $d_2 = h/\sqrt{3}$. Since $d_1 - d_2 = 40$: $h\sqrt{3} - h/\sqrt{3} = 40 \rightarrow h(3-1)/\sqrt{3} = 40 \rightarrow h = 20\sqrt{3} \approx 34.64$ m. Check: d_1=60 m, d_2=20 m, and 60−20=40 m as given.

- From the top of a 100 m cliff, the angles of depression of two boats in line with the cliff's base are 30° and 45°, on the same side. Find the distance between the boats. Worked solution: The nearer boat's horizontal distance is 100/tan45° = 100 m. The farther boat's distance is 100/tan30° = 100√3 m. Distance between boats = 100√3 − 100 = 100(√3−1) ≈ 73.2 m.

> **Quick revision:** tanθ = (vertical rise)/(horizontal distance), from the observer's eye level; add eye height if the ground-level height is asked for. tan30°=1/√3, tan45°=1, tan60°=√3. Two-position problems: write one tan equation per position with the SAME unknown height, and use the known walking distance as the difference of the two horizontal distances. Ladder problems: height = length × sinθ (ladder is the hypotenuse).

Height and Distance Solved Questions

Almost every question in this topic reduces to one right triangle and one of three standard angles — 30°, 45°, or 60° — so the real skill is drawing the triangle correctly and picking the right trig ratio, not memorizing many formulas. This page covers that core approach, two-position (moving observer) problems, and the "flagpole on a tower" two-triangle pattern, with several fully worked examples. The three standard angles and their tangent values (the ratio used most often here): tan 30° = 1/√3, tan 45° = 1, tan 60° = √3. Angle of elevation is measured UP from the horizontal (looking up at something); angle of depression is measured DOWN from the horizontal (looking down at something) — both are measured from the horizontal line, not from the ground or the object. tan(angle) = opposite side / adjacent side, where "opposite" is the height (vertical) and "adjacent" is the horizontal distance from the base — draw the triangle first, label height and distance, then apply tan. For a slanted line like a ladder or a wire (the hypotenuse), use sin or cos instead of tan: cos(angle) = adjacent/hypotenuse when the angle is with the ground, sin(angle) = opposite/hypotenuse. If the angle of elevation changes as an observer walks a known distance toward or away from the base, set up a separate tan equation at each position (both sharing the same unknown height), then solve the two equations together. For a "flagpole/antenna mounted on top of a building" question, where you're given the angles of elevation to the bottom and the top of the flagpole from the same point, draw two right triangles sharing the same horizontal distance, find each height, and subtract to get the flagpole's own length.

Examples

- A pole casts a shadow of length 10√3 m when the sun’s angle of elevation is 30°. Find the height of the pole. Solution: tan(30°) = height / shadow length, so height = shadow length × tan(30°). tan(30°) = 1/√3, so height = 10√3 × (1/√3) = 10 m. Answer: 10 m
- The angle of elevation of the top of a tower from a point 50 m away from its base is 45°. Find the height of the tower. Solution: tan(45°) = height / distance, so height = distance × tan(45°). tan(45°) = 1, so height = 50 × 1 = 50 m. Answer: 50 m
- A ladder leans against a wall, making an angle of 60° with the ground. If the foot of the ladder is 5 m from the wall, find the length of the ladder. Solution: The ladder is the hypotenuse, and 5 m is the side adjacent to the 60° angle: cos(60°) = adjacent / hypotenuse. cos(60°) = 1/2, so hypotenuse = 5 / (1/2) = 10 m. Answer: 10 m
- From the top of a cliff 100 m high, the angle of depression of a boat is 30°. Find the distance of the boat from the base of the cliff. Solution: The angle of depression from the top equals the angle of elevation from the boat (alternate angles), so tan(30°) = height / distance. tan(30°) = 1/√3, so 1/√3 = 100 / distance, giving distance = 100√3 ≈ 173.2 m. Answer: 100√3 m ≈ 173.2 m
- From a point on the ground, the angle of elevation of the top of a tower is 30°. After walking 20 m towards the tower, the angle becomes 60°. Find the height of the tower. Solution: Let height = h, and let the original distance be d. From the first position: tan 30° = h/d, so d = h√3. From the closer position: tan 60° = h/(d − 20), so d − 20 = h/√3. Substitute: h√3 − 20 = h/√3. Multiply through by √3: 3h − 20√3 = h, so 2h = 20√3, giving h = 10√3 ≈ 17.32 m. Answer: 10√3 m ≈ 17.32 m
- A flagpole stands on top of a 20 m tall tower. From a point on the ground, the angle of elevation of the bottom of the flagpole is 30°, and the angle of elevation of the top of the flagpole is 45°. Find the length of the flagpole. Solution: Let the horizontal distance from the point to the tower be d. From the bottom of the flagpole (top of the tower, height 20): tan 30° = 20/d, so d = 20√3. From the top of the flagpole (height 20 + L, where L is the flagpole's length): tan 45° = (20 + L)/d, and since tan 45° = 1, this gives 20 + L = d = 20√3. L = 20√3 − 20 = 20(√3 − 1) ≈ 14.64 m. Answer: 20(√3 − 1) m ≈ 14.64 m

- Practice: A tower casts a shadow of length 20√3 m when the sun's angle of elevation is 30°. Find the height of the tower. (Answer: 20 m)
- Practice: The angle of elevation of the top of a building from a point 40 m away from its base is 45°. Find the height of the building. (Answer: 40 m)
- Practice: A ladder leans against a wall, making an angle of 60° with the ground. If the foot of the ladder is 6 m from the wall, find the length of the ladder. (Answer: 12 m)
- Practice: From a point on the ground, the angle of elevation of the top of a tower is 30°. After walking 40 m towards the tower, the angle becomes 60°. Find the height of the tower. (Answer: 20√3 m ≈ 34.64 m. Same method as the worked example, with 40 m instead of 20 m.)
- Practice: A flagpole stands on top of a 10 m tall tower. The angle of elevation of the flagpole's bottom is 30°, and of its top is 60°, from the same point. Find the flagpole's length. (Answer: 20 m. d = 10√3 from the 30° angle; 10 + L = d√3 = 30, so L = 20.)
- Practice: A wire of length 20 m is attached from the top of a pole to a point on the ground, making an angle of 30° with the ground. Find the height of the pole. (Answer: 10 m. sin 30° = height/20, so height = 20 × 1/2.)
- Practice: From the top of a tower 60 m high, the angle of depression of a car is 45°. Find the distance of the car from the base of the tower. (Answer: 60 m. tan 45° = 60/distance = 1.)

> **Quick revision:** The three standard angles and their tangent values (the ratio used most often here): tan 30° = 1/√3, tan 45° = 1, tan 60° = √3; Angle of elevation is measured UP from the horizontal (looking up at something); angle of depression is measured DOWN from the horizontal (looking down at something) — both are measured from the horizontal line, not from the ground or the object; tan(angle) = opposite side / adjacent side, where "opposite" is the height (vertical) and "adjacent" is the horizontal distance from the base — draw the triangle first, label height and distance, then apply tan; For a slanted line like a ladder or a wire (the hypotenuse), use sin or cos instead of tan: cos(angle) = adjacent/hypotenuse when the angle is with the ground, sin(angle) = opposite/hypotenuse; If the angle of elevation changes as an observer walks a known distance toward or away from the base, set up a separate tan equation at each position (both sharing the same unknown height), then solve the two equations together; For a "flagpole/antenna mounted on top of a building" question, where you're given the angles of elevation to the bottom and the top of the flagpole from the same point, draw two right triangles sharing the same horizontal distance, find each height, and subtract to get the flagpole's own length

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LCM and HCF

LCM and HCF: prime factorization and the product shortcut

HCF (Highest Common Factor) is the largest number that divides every given number exactly. LCM (Lowest Common Multiple) is the smallest number every given number divides into exactly. Prime factorization method: break each number into prime factors. HCF = product of the common prime factors, each at its lowest power across the numbers. LCM = product of all prime factors (common and uncommon), each at its highest power. Useful shortcut for TWO numbers a and b: HCF(a, b) × LCM(a, b) = a × b, letting one be found directly from the other without refactorizing. For "largest number that divides X, Y and Z leaving the same remainder R" questions, find the HCF of (X − R), (Y − R) and (Z − R) instead of X, Y, Z directly.

A critical caveat: the HCF × LCM = product shortcut works ONLY for exactly two numbers - it does not generalize to three or more. For 4, 6, and 8: HCF = 2 and LCM = 24, so HCF × LCM = 48, but the actual product 4×6×8 = 192. These are not equal, and relying on the two-number shortcut for a three-number question is a common, costly mistake.

When numbers are large and hard to factorize by inspection, the Euclidean algorithm (successive division) finds the HCF without needing prime factorization at all: divide the larger number by the smaller, then repeatedly divide the previous divisor by the remainder, until the remainder is 0 - the last non-zero divisor is the HCF. Once the HCF of two large numbers is found this way, their LCM can still be recovered instantly via LCM = (a×b)/HCF.

LCM and HCF also extend to fractions: HCF of fractions = HCF(numerators) / LCM(denominators), and LCM of fractions = LCM(numerators) / HCF(denominators) - note the denominators swap which combining rule they use compared to the numerators. Finally, two numbers with no common factor other than 1 are called co-prime, and for co-prime numbers specifically, the LCM is simply their product (since there's no overlap to divide out).

Examples

- $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$: HCF = $2^1 \times 3^1 = 6$, LCM = $2^2 \times 3^2 = 36$; check HCF × LCM = 216 = 12 × 18.
- Two numbers in the ratio 3:4 have LCM 48: writing them as 3x and 4x, LCM = 12x = 48 gives x = 4, so the numbers are 12 and 16 with HCF = 4.
- The largest number dividing 70 and 125 leaving remainders 5 and 8 is HCF(70−5, 125−8) = HCF(65, 117) = 13; check 70÷13 = 5 r5 and 125÷13 = 9 r8.
- $24 = 2^3 \times 3$, $36 = 2^2 \times 3^2$, $60 = 2^2 \times 3 \times 5$: HCF (common to all three) = $2^2 \times 3 = 12$, LCM = $2^3 \times 3^2 \times 5 = 360$ - note HCF×LCM = 4320 is nowhere close to the actual product 24×36×60 = 51,840, since the two-number shortcut doesn't apply with three numbers.
- For 4, 6, and 8: HCF = 2 and LCM = 24, so HCF×LCM = 48 - but the actual product 4×6×8 = 192, confirming the shortcut HCF×LCM=product only ever holds for exactly two numbers.
- To find HCF(414, 662) by successive division: 662 = 1×414+248, 414 = 1×248+166, 248 = 1×166+82, 166 = 2×82+2, 82 = 41×2+0 - the last non-zero remainder is 2, so HCF(414,662) = 2, and LCM = (414×662)/2 = 137,034.
- HCF and LCM of 2/3 and 4/9: HCF = HCF(2,4)/LCM(3,9) = 2/9, and LCM = LCM(2,4)/HCF(3,9) = 4/3; check (2/3)÷(2/9) = 3 and (4/9)÷(2/9) = 2, both whole numbers as required.
- 7 and 9 share no common factor other than 1 (they are co-prime), so their LCM is simply their product: LCM(7,9) = 63, with HCF = 1.

> **Quick revision:** HCF = common prime factors at lowest power; LCM = all prime factors at highest power. HCF × LCM = a × b holds ONLY for two numbers, never three or more. Same-remainder problems: HCF of (numbers minus the remainder). Large numbers: use successive (Euclidean) division. Fractions: HCF = HCF(num)/LCM(den), LCM = LCM(num)/HCF(den). Co-prime numbers: LCM = their product.

LCM and HCF Solved Questions

LCM and HCF questions reward knowing the prime-factorization method cold, plus the shortcuts connecting the two and the standard "remainder" variants exams favour. This page covers the core method, the two classic remainder patterns, the LCM application for repeating events (like bells ringing together), and several fully worked examples. HCF (Highest Common Factor) is the largest number that divides every given number exactly. LCM (Lowest Common Multiple) is the smallest number that every given number divides into exactly. Prime factorization method: break each number into prime factors. HCF = product of the common prime factors, each at its LOWEST power across the numbers. LCM = product of all prime factors (common and uncommon), each at its HIGHEST power. Useful shortcut for two numbers a and b: HCF(a, b) × LCM(a, b) = a × b. This lets you find one directly from the other without refactorizing. If a and b are co-prime (HCF = 1), this simplifies further: LCM = a × b directly. For "largest number that divides X, Y and Z leaving the same remainder R" questions, find the HCF of (X − R), (Y − R) and (Z − R) instead of X, Y, Z directly. "Smallest number divisible by a, b and c" = LCM(a, b, c). "Smallest number that leaves the SAME remainder R when divided by each of a, b and c" = LCM(a, b, c) + R. "Smallest number that, when divided by a, b and c, leaves remainders each exactly k less than their own divisor" (check this holds for every divisor first) = LCM(a, b, c) − k. Events that repeat at fixed intervals and need to coincide again (bells ringing together, runners starting together, lights changing together) is an LCM application: the time until they next coincide is the LCM of their individual intervals.

Examples

- Find the HCF and LCM of 12 and 18. Solution: $12 = 2^2 \times 3$. $18 = 2 \times 3^2$. HCF = common factors at their lowest power = $2^1 \times 3^1 = 6$. LCM = all factors at their highest power = $2^2 \times 3^2 = 4 \times 9 = 36$. Check: HCF × LCM = 6 × 36 = 216, and 12 × 18 = 216 — matches. Answer: HCF = 6, LCM = 36
- Find the HCF and LCM of 24, 36, and 60. Solution: $24 = 2^3 \times 3$. $36 = 2^2 \times 3^2$. $60 = 2^2 \times 3 \times 5$. HCF = common factors (2 and 3 appear in all three; 5 doesn't) at their lowest power across all three: $2^2 \times 3^1 = 4 \times 3 = 12$. LCM = all factors at their

highest power: $2^3 \times 3^2 \times 5^1 = 8 \times 9 \times 5 = 360$. Answer: HCF = 12, LCM = 360

- Two numbers are in the ratio 3 : 4 and their LCM is 48. Find the HCF and the two numbers. Solution: Let the numbers be 3x and 4x. Since 3 and 4 share no common factor, x itself is the HCF, and the LCM is $3 \times 4 \times x = 12x$. $12x = 48 \rightarrow x = 4$. The numbers are $3 \times 4 = 12$ and $4 \times 4 = 16$. HCF = x = 4. Check: HCF(12, 16) = 4 and LCM(12, 16) = 48 — matches. Answer: HCF = 4, numbers are 12 and 16
- Find the largest number that divides 70 and 125, leaving remainders 5 and 8 respectively. Solution: The required number is the HCF of (70 – 5) and (125 – 8), i.e., HCF(65, 117). $65 = 5 \times 13$. $117 = 3^2 \times 13$. HCF = 13. Check: 70 ÷ 13 = 5 remainder 5. 125 ÷ 13 = 9 remainder 8 — both match. Answer: 13
- Find the smallest number which, when divided by 12, 16 and 24, leaves a remainder of 7 in each case. Solution: First find LCM(12, 16, 24): $12 = 2^2 \times 3$, $16 = 2^4$, $24 = 2^3 \times 3 \rightarrow$ LCM $= 2^4 \times 3 = 48$. Smallest number leaving the same remainder R = LCM + R = 48 + 7 = 55. Answer: 55
- Find the smallest number which, when divided by 20, 25 and 30, leaves remainders 17, 22 and 27 respectively. Solution: Check the "k less than divisor" pattern: 20–17=3, 25–22=3, 30–27=3 — consistent, so k = 3. LCM(20, 25, 30): $20 = 2^2 \times 5$, $25 = 5^2$, $30 = 2 \times 3 \times 5 \rightarrow$ LCM $= 2^2 \times 3 \times 5^2 = 300$. Required number = LCM – k = 300 – 3 = 297. Check: 297 ÷ 20 = 14 remainder 17; 297 ÷ 25 = 11 remainder 22; 297 ÷ 30 = 9 remainder 27 — all match. Answer: 297
- Three bells ring at intervals of 15, 20 and 30 minutes respectively. If they all ring together at 9:00 AM, when will they next ring together? Solution: LCM(15, 20, 30): $15 = 3 \times 5$, $20 = 2^2 \times 5$, $30 = 2 \times 3 \times 5 \rightarrow$ LCM $= 2^2 \times 3 \times 5 = 60$. They next ring together after 60 minutes, i.e. at 10:00 AM. Answer: 10:00 AM
- Practice: Find the HCF and LCM of 15 and 20. (Answer: HCF = 5, LCM = 60)
- Practice: Find the HCF and LCM of 8, 12, and 20. (Answer: HCF = 4, LCM = 120)
- Practice: The HCF of two numbers is 6 and their LCM is 90. If one of the numbers is 18, find the other. (Answer: 30. HCF × LCM = product of numbers = 540; 540/18.)
- Practice: Find the smallest number which, when divided by 10, 15 and 20, leaves a remainder of 4 in each case. (Answer: 64. LCM(10,15,20) = 60; 60 + 4.)
- Practice: Find the smallest number which, when divided by 15, 20 and 25, leaves remainders 11, 16 and 21 respectively. (Answer: 296. k = 4 in each case (15–11=20–16=25–21=4); LCM(15,20,25) = 300; 300 – 4.)
- Practice: Three bells ring at intervals of 6, 8 and 12 minutes. If they ring together at 12:00 noon, when will they next ring together? (Answer: 12:24 PM. LCM(6,8,12) = 24 minutes.)
- Practice: Find the LCM of 7 and 11. (Answer: 77. 7 and 11 are co-prime (HCF = 1), so LCM = 7 × 11 directly.)

> **Quick revision:** HCF (Highest Common Factor) is the largest number that divides every given number exactly. LCM (Lowest Common Multiple) is the smallest number that every given number divides into exactly; Prime factorization method: break each number into prime factors. HCF = product of the common prime factors, each at its LOWEST power across the numbers. LCM = product of all prime factors (common and uncommon), each at its HIGHEST power; Useful shortcut for two numbers a and b: HCF(a, b) × LCM(a, b) = a × b. This lets you find one directly from the other without refactorizing. If a and b are co-prime (HCF = 1), this simplifies further: LCM = a × b directly; For "largest number that divides X, Y and Z leaving the same remainder R" questions, find the HCF of (X – R), (Y – R) and (Z – R) instead of X, Y, Z directly; "Smallest number divisible by a, b and c" = LCM(a, b, c). "Smallest number that leaves the SAME remainder R when divided by each of a, b and c" = LCM(a, b, c) + R; "Smallest number that, when divided by a, b and c, leaves remainders each exactly k less than their own divisor" (check this holds for every divisor first) = LCM(a, b, c) – k; Events that repeat at fixed intervals and need to coincide again (bells ringing together, runners starting together, lights changing together) is an LCM application: the time until they next coincide is the LCM of their individual intervals

HCF and LCM of Fractions Solved Questions

This isn't a new idea — it's the same HCF/LCM method from whole numbers, applied to the numerators and denominators separately — but it only works correctly if you reduce every fraction to its lowest terms FIRST. This page covers the formula, that essential reduce-first step (with a worked example showing why skipping it can silently give a wrong answer), and a verification check, with several fully worked examples. HCF of fractions = HCF of the numerators / LCM of the denominators — provided every fraction has first been written in its lowest terms (numerator and denominator sharing no common factor). LCM of fractions = LCM of the numerators / HCF of the

denominators — under the same lowest-terms condition. Always reduce every given fraction to lowest terms FIRST, before taking the HCF of the numerators or the LCM of the denominators. Skipping this can silently give a wrong answer: for 2/4 and 3/6 (both equal to 1/2), the raw unreduced formula gives HCF(2,3)/LCM(4,6) = 1/12 — which is NOT the correct HCF, since 1/2 itself is a far larger value that both fractions are exact multiples of. Reducing first (1/2 and 1/2) correctly gives HCF = 1/2. Simplifying the FINAL answer (if it isn't already in lowest terms) is a separate, optional last step for a cleaner answer — it does not substitute for reducing the inputs first, and won't reliably fix an error introduced by skipping that step. Check your answer: an HCF of fractions must divide evenly into every given fraction (each original ÷ your HCF should be a whole number); an LCM of fractions must itself be evenly divisible by every given fraction (your LCM ÷ each original should be a whole number). The whole process mirrors whole-number HCF/LCM exactly — reduce each fraction, apply HCF/LCM to the numerators and denominators as described, then simplify the result if needed. The only new skill is handling a numerator and a denominator together instead of a single number.

Examples

- Find the HCF of 2/3 and 4/9. Solution: Both fractions are already in lowest terms. HCF of numerators: HCF(2, 4) = 2. LCM of denominators: LCM(3, 9) = 9. HCF of fractions = 2/9. Answer: 2/9
- Find the LCM of 2/3 and 4/9. Solution: Both fractions are already in lowest terms. LCM of numerators: LCM(2, 4) = 4. HCF of denominators: HCF(3, 9) = 3. LCM of fractions = 4/3. Answer: 4/3
- Find the HCF of 4/5, 8/15, and 12/25. Solution: All three fractions are already in lowest terms. HCF of numerators: HCF(4, 8, 12) = 4. LCM of denominators: LCM(5, 15, 25) = 75. HCF of fractions = 4/75. Answer: 4/75
- Find the LCM of 4/5, 8/15, and 12/25, and verify it. Solution: LCM of numerators: LCM(4, 8, 12) = 24. HCF of denominators: HCF(5, 15, 25) = 5. LCM of fractions = 24/5. Verify: (24/5) ÷ (4/5) = 6 ✓, (24/5) ÷ (8/15) = 9 ✓, (24/5) ÷ (12/25) = 10 ✓ — all whole numbers, as required for a genuine LCM. Answer: 24/5
- Find the HCF of 2/4 and 3/6. Solution: Neither fraction is in lowest terms: 2/4 = 1/2 and 3/6 = 1/2. Reduce both first. Now apply the formula to the reduced fractions: HCF(1, 1) = 1, LCM(2, 2) = 2. HCF = 1/2. Using the raw, unreduced numbers instead would have wrongly given HCF(2,3)/LCM(4,6) = 1/12 — this is why reducing first matters. Answer: 1/2
- Find the HCF of 6/7, 12/21, and 18/35 (notice 12/21 is not in lowest terms). Solution: Reduce 12/21 first: HCF(12,21) = 3, so 12/21 = 4/7. The other two fractions, 6/7 and 18/35, are already in lowest terms. Now apply the formula to 6/7, 4/7, 18/35: HCF of numerators = HCF(6, 4, 18) = 2. LCM of denominators = LCM(7, 7, 35) = 35. HCF of fractions = 2/35. Answer: 2/35
- Find the LCM of 3/6 and 5/10. Solution: Reduce both first: 3/6 = 1/2 and 5/10 = 1/2 — they're actually the same value. LCM of numerators: LCM(1, 1) = 1. HCF of denominators: HCF(2, 2) = 2. LCM = 1/2 (the LCM of two equal values is just that value itself). Answer: 1/2
- Practice: Find the HCF of 3/4 and 9/10. (Answer: 3/20)
- Practice: Find the LCM of 3/4 and 9/10. (Answer: 9/2)
- Practice: Find the HCF of 6/7, 12/21, and 18/35. (Answer: 2/35. Reduce 12/21 to 4/7 first (dividing by their HCF, 3); then HCF(6,4,18)/LCM(7,7,35).)
- Practice: Find the HCF of 4/8 and 6/12. (Answer: 1/2. Both reduce to 1/2 first.)
- Practice: Find the LCM of 2/4 and 6/8. (Answer: 3/2. Reduce first to 1/2 and 3/4; LCM(1,3)/HCF(2,4) = 3/2.)
- Practice: Find the HCF of 8/10, 4/5, and 12/20. (Answer: 1/5. Reduce first to 4/5, 4/5, 3/5; HCF(4,4,3)/LCM(5,5,5) = 1/5.)
- Practice: Find the LCM of 5/6 and 7/9. (Answer: 35/3. Both already in lowest terms: LCM(5,7)/HCF(6,9).)

Quick revision: HCF of fractions = HCF of the numerators / LCM of the denominators — provided every fraction has first been written in its lowest terms (numerator and denominator sharing no common factor); LCM of fractions = LCM of the numerators / HCF of the denominators — under the same lowest-terms condition; Always reduce every given fraction to lowest terms FIRST, before taking the HCF of the numerators or the LCM of the denominators. Skipping this can silently give a wrong answer: for 2/4 and 3/6 (both equal to 1/2), the raw unreduced formula gives HCF(2,3)/LCM(4,6) = 1/12 — which is NOT the correct HCF, since 1/2 itself is a far larger value that both fractions are exact multiples of. Reducing first (1/2 and 1/2) correctly gives HCF = 1/2; Simplifying the FINAL answer (if it isn't already in lowest terms) is a separate, optional last step for a cleaner answer — it does not substitute for reducing the inputs first, and won't reliably fix an error introduced by skipping that step; Check your answer: an HCF of fractions must divide evenly into every given fraction (each original ÷ your HCF should be a whole number); an LCM of fractions must itself be evenly divisible by every given fraction (your LCM ÷ each original should be a whole number); The whole process mirrors whole-number HCF/LCM exactly — reduce each fraction, apply HCF/LCM to the numerators and denominators as described, then simplify the result if needed. The only new skill is handling a numerator and a denominator together instead of a single number

Free practice questions: parikshasaathi.com/q/lcm-and-hcf/

Mensuration: 2D

Mensuration: 2D — Concepts and Applications

Perimeter measures a figure's boundary length, while area measures the enclosed region in square units. A rectangle has area lb and perimeter 2(l + b), a square has area s^2 and perimeter 4s, and a triangle has area base × perpendicular height/2. Parallelogram area is base × height, rhombus area is half the product of its diagonals, and trapezium area is half the sum of its parallel sides times their separation. A regular polygon's perimeter is side count × side length, and its area is half its perimeter times its apothem. A circle has circumference $2\pi r$ and area πr^2; a sector takes angle/360 of the circle's area and circumference for its area and arc length. Split combined figures into non-overlapping parts or subtract explicitly removed regions, keeping units and the stated value of π consistent.

For a sector, both its area and its arc length use the SAME fraction of the full circle (angle/360°) - area = (angle/360°) × πr^2 and arc length = (angle/360°) × $2\pi r$ - so finding one often means the other is just a matter of swapping in the full-circle area or circumference formula.

A regular hexagon has a particularly clean area formula worth remembering separately: $(3\sqrt{3}/2) \times \text{side}^2$, which comes from splitting it into 6 equilateral triangles - useful since the general apothem-based polygon formula requires computing the apothem first, while this shortcut skips that step entirely for hexagons specifically.

Examples

- Find the area of a square of side 6 cm, in cm^2. Worked solution: A square has four equal sides and right angles. Its area is 6 × 6 = 36 cm^2.
- A rhombus has diagonals 12 and 16 cm. Find its area in cm^2. Worked solution: Area is half the diagonal product: 12 × 16/2 = 96 cm^2.
- A rectangle 22 cm long and 28 cm wide has a semicircle of radius 14 cm attached externally along one width. Interiors do not overlap. Find the combined area in cm^2, using π = 22/7. Worked solution: Rectangle area is 22 × 28 = 616. Semicircle area is (22/7) × $14^2/2$ = 308. Add the non-overlapping areas to obtain 924 cm^2.
- A sector of a circle with radius 21 cm has a central angle of 60°. Find its area and arc length, using π = 22/7. Worked solution: Area = (60/360) × (22/7) × 21^2 = (1/6) × 1386 = 231 cm^2. Arc length = (60/360) × 2 × (22/7) × 21 = (1/6) × 132 = 22 cm.
- Find the area of a trapezium with parallel sides 10 cm and 14 cm, and height 6 cm. Worked solution: Area = ½ × (sum of parallel sides) × height = ½ × (10+14) × 6 = ½ × 24 × 6 = 72 cm^2.
- Find the area of a regular hexagon with side 6 cm, leaving the answer in terms of √3. Worked solution: Using the hexagon shortcut, area = $(3\sqrt{3}/2) \times 6^2 = (3\sqrt{3}/2) \times 36 = 54\sqrt{3}$ cm^2 - obtained directly without first computing the apothem.

Quick revision: Rectangle: area=lb, perimeter=2(l+b). Square: area=s^2, perimeter=4s. Triangle: area=½×base×height. Rhombus: area=½×d_1×d_2. Trapezium: area=½×(sum of parallel sides)×height. Circle: circumference=2πr, area=πr^2. Sector: both area and arc length use the SAME fraction (angle/360°) of the full circle's formula. Regular hexagon shortcut: area=$(3\sqrt{3}/2)\times side^2$.

Mensuration (Area & Perimeter)

Mensuration questions are mostly about picking the right formula for the shape in front of you and plugging in carefully. This page covers rectangles, circles, triangles (including Heron's formula when only the three sides are known), parallelograms, rhombuses, trapeziums and circular sectors, with several fully worked examples. Perimeter is the distance around a shape; area is the space it covers. Rectangle: Perimeter = 2(length + breadth), Area = length × breadth. Square (a rectangle with equal sides): Perimeter = 4 × side, Area = $side^2$. Circle: Circumference = 2πr, Area = πr^2. Use π ≈ 22/7 when the radius is a multiple of 7 (it cancels cleanly); otherwise use π ≈ 3.14. Triangle: Area = (1/2) × base × height. For a right-angled triangle specifically, the two legs meeting at the right angle can be used directly as base and height. When only the three sides a, b, c are known (no height), use Heron's formula instead: Area = $\sqrt{(s(s-a)(s-b)(s-c))}$, where s = (a+b+c)/2 is the semi-perimeter. Parallelogram: Area = base × height, where height is the perpendicular distance between the pair of parallel sides, not the length of the slanted side. Rhombus (a parallelogram with all sides equal): Area = (1/2) × d_1 × d_2, using the two diagonals. Trapezium: Area = (1/2) × (sum of the two parallel sides) × height. Equilateral triangle (a special case where Heron's formula simplifies): Area = $(\sqrt{3}/4) \times side^2$. Sector of a circle (a "slice" of angle θ degrees out of the full 360°): Area = (θ/360) × πr^2, and the arc length = (θ/360) × 2πr. A semicircle is just the θ = 180° case of both formulas. For "path around a field" or "border" questions, find the outer shape's area and the inner shape's area separately, then subtract — don't try to compute the path's area directly as one piece.

Examples

- Find the area and perimeter of a rectangle with length 15 m and breadth 8 m. Solution: Area = length × breadth = 15 × 8 = 120 m^2 Perimeter = 2(length + breadth) = 2(15 + 8) = 2 × 23 = 46 m Answer: Area = 120 m^2, Perimeter = 46 m
- A circular park has a radius of 14 m. Find its area. (Use π = 22/7.) Solution: Area = πr^2 = (22/7) × 14 × 14 14 is a multiple of 7, so it cancels cleanly: (22/7) × 14 = 44, then 44 × 14 = 616 Answer: 616 m^2
- Find the area of a right-angled triangle with base 12 cm and height 9 cm. Also find the length of its hypotenuse. Solution: Area = (1/2) × base × height = (1/2) × 12 × 9 = 54 cm^2 Hypotenuse = $\sqrt{(base^2 + height^2)} = \sqrt{(12^2 + 9^2)} = \sqrt{(144 + 81)} = \sqrt{225}$ = 15 cm Answer: Area = 54 cm^2, Hypotenuse = 15 cm
- A rectangular garden is 20 m long and 15 m wide. A path 2.5 m wide is built all around it, on the outside. Find the area of the path. Solution: The path adds 2.5 m on every side, so the outer rectangle is (20 + 2×2.5) by (15 + 2×2.5) = 25 m by 20 m. Outer area = 25 × 20 = 500 m^2. Garden (inner) area = 20 × 15 = 300 m^2. Area of the path = outer area − inner area = 500 − 300 = 200 m^2 Answer: 200 m^2
- Find the area of a parallelogram with base 12 cm and the perpendicular height between its parallel sides equal to 7 cm. Solution: Area = base × height = 12 × 7 = 84 cm^2 Answer: 84 cm^2
- The diagonals of a rhombus are 16 cm and 12 cm. Find its area. Solution: Area = (1/2) × d_1 × d_2 = (1/2) × 16 × 12 = 96 cm^2 Answer: 96 cm^2
- A trapezium has parallel sides of length 10 cm and 14 cm, and the perpendicular distance between them is 6 cm. Find its area. Solution: Area = (1/2) × (sum of parallel sides) × height = (1/2) × (10 + 14) × 6 = (1/2) × 24 × 6 = 72 cm^2 Answer: 72 cm^2
- A triangle has sides 13 cm, 14 cm and 15 cm. Find its area using Heron's formula. Solution: Semi-perimeter s = (13 + 14 + 15) / 2 = 21. Area = $\sqrt{(s(s-a)(s-b)(s-c))} = \sqrt{(21 \times (21-13) \times (21-14) \times (21-15))} = \sqrt{(21 \times 8 \times 7 \times 6)} = \sqrt{7{,}056}$ = 84 cm^2 Answer: 84 cm^2
- Practice: Find the area and perimeter of a rectangle with length 18 m and breadth 10 m. (Answer: Area = 180 m^2, Perimeter = 56 m)
- Practice: A circular garden has a radius of 21 m. Find its area. (Use π = 22/7.) (Answer: 1,386 m^2)
- Practice: Find the area of a right-angled triangle with base 10 cm and height 24 cm. Also find its hypotenuse. (Answer: Area = 120 cm^2, Hypotenuse = 26 cm)

- Practice: Find the area of a parallelogram with base 15 cm and height 8 cm. (Answer: 120 cm^2. Area = base × height.)
- Practice: A rhombus has diagonals of 10 cm and 24 cm. Find its area. (Answer: 120 cm^2. Area = (1/2) × 10 × 24.)
- Practice: A trapezium has parallel sides 8 cm and 12 cm with a height of 5 cm. Find its area. (Answer: 50 cm^2. Area = (1/2) × (8+12) × 5.)
- Practice: Find the area of an equilateral triangle with side 8 cm. (Answer: $16\sqrt{3}$ $cm^2 \approx 27.7$ cm^2. Area = $(\sqrt{3}/4) \times 8^2$.)
- Practice: Find the area and arc length of a sector of radius 21 cm with a central angle of 60°. (Use $\pi = 22/7$.) (Answer: Area = 231 cm^2, Arc length = 22 cm. Area = (60/360) × (22/7) × 21^2; arc = (60/360) × 2 × (22/7) × 21.)

> **Quick revision:** Perimeter is the distance around a shape; area is the space it covers. Rectangle: Perimeter = 2(length + breadth), Area = length × breadth. Square (a rectangle with equal sides): Perimeter = 4 × side, Area = $side^2$; Circle: Circumference = $2\pi r$, Area = πr^2. Use $\pi \approx 22/7$ when the radius is a multiple of 7 (it cancels cleanly); otherwise use $\pi \approx 3.14$; Triangle: Area = (1/2) × base × height. For a right-angled triangle specifically, the two legs meeting at the right angle can be used directly as base and height. When only the three sides a, b, c are known (no height), use Heron's formula instead: Area = $\sqrt{(s(s-a)(s-b)(s-c))}$, where $s = (a+b+c)/2$ is the semi-perimeter; Parallelogram: Area = base × height, where height is the perpendicular distance between the pair of parallel sides, not the length of the slanted side. Rhombus (a parallelogram with all sides equal): Area = (1/2) × d_1 × d_2, using the two diagonals; Trapezium: Area = (1/2) × (sum of the two parallel sides) × height; Equilateral triangle (a special case where Heron's formula simplifies): Area = $(\sqrt{3}/4) \times side^2$; Sector of a circle (a "slice" of angle θ degrees out of the full 360°): Area = (θ/360) × πr^2, and the arc length = (θ/360) × $2\pi r$. A semicircle is just the θ = 180° case of both formulas; For "path around a field" or "border" questions, find the outer shape's area and the inner shape's area separately, then subtract — don't try to compute the path's area directly as one piece

Free practice questions: parikshasaathi.com/q/mensuration-2d/

Mensuration: 3D

Mensuration: 3D — Concepts and Applications

Volume is measured in cubic units and surface area in square units, with only the exposed faces counted for a combined or open solid. A prism has volume base area × height; a cuboid has volume lbh and total surface area 2(lb + bh + hl), while a cube has volume s^3 and surface area $6s^2$. A cylinder has volume $\pi r^2 h$ and total surface area $2\pi r(h + r)$; a cone has volume $\pi r^2 h/3$ and lateral area πrl, where l is slant height. A sphere has volume $4\pi r^3/3$ and surface area $4\pi r^2$, while a hemisphere has half that volume, curved area $2\pi r^2$ and total area $3\pi r^2$ including its circular base. A regular pyramid has volume base area × perpendicular height/3 and lateral area half the base perimeter times the face slant height. For joined solids add non-overlapping volumes, remove hidden shared faces from exposed area, and conserve volume in melting and recasting when no material is lost.

For a cylinder, the total surface area splits naturally into three parts: two circular ends ($2\pi r^2$ total) plus the curved side ($2\pi rh$) - writing it as $2\pi r(h+r)$ is just a factored version of $2\pi rh + 2\pi r^2$, so either form gives the same answer.

A hemisphere has THREE distinct area quantities worth keeping separate: the curved (dome) surface alone is $2\pi r^2$, adding the flat circular base gives the total surface area $3\pi r^2$, and the volume (a completely different kind of quantity, measured in cubic units) is $(2/3)\pi r^3$ - mixing these up is a common source of error.

Examples

- A closed cube has edge 5 cm. Find its total surface area in cm^2. Worked solution: Each of 6 square faces has area 5^2 = 25. Total surface area is 6 × 25 = 150 cm^2.
- A closed right circular cone has radius 6 cm and height 8 cm. Find its total surface area in cm^2, using $\pi = 22/7$. Worked solution: Slant height = $\sqrt{(6^2 + 8^2)}$ = 10 cm. Lateral area plus base area is $\pi r(l + r) = (22/7) \times 6 \times (10 + 6) = 2112/7$ cm^2.
- A solid sphere of radius 4 cm is melted and recast into solid spheres each of radius 2 cm, with no material loss. How many small spheres are made? Worked solution: Volume is proportional to the cube of radius. The count is $(4/2)^3 = 2^3$ = 8.
- A closed cylinder has radius 7 cm and height 10 cm. Find its volume, curved surface area, and total surface area, using $\pi = 22/7$. Worked solution: Volume = $\pi r^2 h = (22/7) \times 49 \times 10 = 1540$ cm^3. Curved surface area = $2\pi rh = 2 \times (22/7) \times 7 \times 10$ =

440 cm². Total surface area = 2πr(h+r) = 2×(22/7)×7×17 = 748 cm².

- A hemisphere has radius 6 cm. Find its volume, curved surface area, and total surface area, leaving answers in terms of π. Worked solution: Volume = (2/3)πr³ = (2/3)π×216 = 144π cm³. Curved surface area = 2πr² = 2π×36 = 72π cm². Total surface area = 3πr² = 3π×36 = 108π cm² - three genuinely different quantities, not to be confused with each other.

> **Quick revision:** Cuboid: volume=lbh, TSA=2(lb+bh+hl). Cube: volume=s³, TSA=6s². Cylinder: volume=πr²h, CSA=2πrh, TSA=2πr(h+r)=CSA+2 circular ends. Cone: volume=πr²h/3, lateral area=πrl (l=slant height). Sphere: volume=4πr³/3, SA=4πr². Hemisphere: volume=(2/3)πr³, curved area=2πr², total area=3πr² - three distinct quantities. Melting/recasting conserves volume, not surface area.

Mixture and Alligation

Mixture and Alligation — Concepts and Applications

For additive quantities, a mixture's unit price or concentration is the total value or ingredient quantity divided by the total mixture quantity. Alligation gives the low:high quantity ratio as (high value – target):(target – low value) when the target lies strictly between the two values. Adding water changes total volume but leaves the amount of the non-water ingredient unchanged. Removing a well-mixed portion removes each ingredient in its current proportion. If a fixed portion is removed and replaced repeatedly with pure water while total volume stays V, the original ingredient remaining after k operations is its starting amount times (1 – removed/V)^k.

When two already-mixed solutions (not raw ingredients) are combined, find the total amount of the ingredient of interest in each solution first (volume × concentration), add those amounts together, and divide by the combined total volume - this is really the same weighted-average idea as combining prices, just applied to concentrations.

The repeated-dilution formula is worth working through numerically at least once: a container that starts with some quantity, has a fixed amount removed and replaced with water, and this is repeated several times, loses the original substance geometrically - each operation multiplies what remains by the same retained fraction, not a fixed amount, which is why the formula uses a power rather than simple subtraction.

Examples

- 11 kg of grain priced at ₹30/kg is mixed with 21 kg priced at ₹50/kg. With no loss or extra cost, find the mixture cost per kg in rupees. Worked solution: Total cost = 11 × 30 + 21 × 50 = ₹1380. Total mass is 32 kg, so cost per kg is ₹43.125.
- 33 litres of a 20% solute solution receives pure solute until its concentration is 36%. Assume liquid volumes add. How many litres of pure solute are added? Worked solution: Let added solute be x. Then (0.20 × 33 + x)/(33 + x) = 0.36. Rearranging gives 0.64x = 0.16 × 33, hence x = 8.25 litres.
- Mix 11 kg at ₹20/kg, 16 kg at ₹40/kg and 21 kg at ₹60/kg, with no loss or extra costs. Find the average cost per kg in rupees. Worked solution: The three costs total ₹2120; total mass is 48 kg. Dividing gives ₹265/6 per kg.
- A container has 40 litres of milk. 8 litres are removed and replaced with water, and this is repeated 2 more times (3 operations total). Find the milk remaining. Worked solution: Remaining milk = 40 × (1 – 8/40)³ = 40 × 0.8³ = 40 × 0.512 = 20.48 litres - each operation retains 80% of whatever milk remained, compounding geometrically.
- 10 litres of a 20% alcohol solution is mixed with 15 litres of a 50% alcohol solution. Find the resulting concentration. Worked solution: Total alcohol = 10×0.20 + 15×0.50 = 2 + 7.5 = 9.5 litres. Total volume = 25 litres. Concentration = 9.5/25 × 100 = 38%.
- In what ratio should tea worth ₹60/kg be mixed with tea worth ₹90/kg so the mixture costs ₹80/kg? Worked solution: By alligation, ratio = (90–80):(80–60) = 10:20 = 1:2 - one part of the cheaper tea for every two parts of the dearer tea.

> **Quick revision:** Mixture value/concentration = weighted total/total quantity; alligation ratio = (high – mean):(mean – low). Combining two solutions: add the actual ingredient amounts (volume × concentration) before dividing by total volume. Retained fraction after n replacement operations = (1 – removed/total)^n - a geometric decay, not a linear one.

Mixture and Alligation Solved Questions

Alligation is a shortcut for finding the ratio to mix two things of different value so the mixture comes out at a target average — it's really just the weighted-average formula worked backwards. This page covers the core rule, the

repeated-dilution ("remove and replace") pattern, mixing more than two components, and several fully worked examples. Alligation rule: to mix a cheaper item (price C) and a dearer item (price D) so the mixture's mean price is M, the required ratio is cheaper : dearer = (D − M) : (M − C). This only makes sense when M is strictly between C and D — if the target mean is outside that range, the mixture as described is impossible. This is just the weighted-average equation rearranged — you can always sanity-check an answer by computing the weighted average directly from the ratio you found and confirming it equals the target mean. The same rule works for anything with a "concentration" — milk-water mixtures, alloy compositions, even mixing two classes' average marks — not just prices. For "cost price vs selling price" alligation questions (like diluting milk with free water to make a profit), treat water's cost as ₹0 and work out the mixture's effective cost price from the profit percentage first, before applying the ratio formula. For mixing three or more components at once, the two-term alligation shortcut does not directly apply — fall back to the weighted-average definition: total value of all components ÷ total quantity. Repeated dilution ("remove and replace"): if a container holds volume V of a pure liquid, and each time x units are withdrawn and replaced with water, then after n such operations the pure liquid remaining = $V \times (1 - x/V)^n$. This is a geometric decay, not a linear one — don't just subtract nx.

Examples

- In what ratio should tea worth ₹60/kg be mixed with tea worth ₹80/kg so that the mixture is worth ₹65/kg? Solution: Using the alligation rule: cheaper : dearer = (80 − 65) : (65 − 60) = 15 : 5 = 3 : 1. Check: weighted average = (3 × 60 + 1 × 80) / 4 = (180 + 80) / 4 = 260/4 = 65 — matches. Answer: 3 : 1 (₹60/kg tea : ₹80/kg tea)
- A trader mixes 20 kg of rice costing ₹40/kg with 30 kg of rice costing ₹60/kg. Find the average price of the mixture. Solution: Total cost = (20 × 40) + (30 × 60) = 800 + 1,800 = ₹2,600 Total quantity = 20 + 30 = 50 kg Average price = 2,600 / 50 = ₹52/kg Answer: ₹52/kg
- A mixture of 60 litres contains milk and water in the ratio 2 : 1. How much water must be added to make the ratio of milk to water 1 : 2? Solution: Milk = (2/3) × 60 = 40 L, Water = (1/3) × 60 = 20 L. Adding water doesn't change the milk, so if x litres of water are added: 40 / (20 + x) = 1 / 2 80 = 20 + x → x = 60 litres. Check: 40 : (20 + 60) = 40 : 80 = 1 : 2. Answer: 60 litres
- A milkman mixes water (free) with milk and sells the mixture at the same price as pure milk, thereby gaining 20%. Find the ratio of water to milk in the mixture. Solution: Let the price of pure milk (and the selling price of the mixture) = ₹6/litre. A 20% gain means the mixture's own cost price = SP ÷ 1.2 = 6 ÷ 1.2 = ₹5/litre. Apply alligation with water (₹0) as the "cheaper" item and milk (₹6) as the "dearer" item, mean = ₹5: water : milk = (6 − 5) : (5 − 0) = 1 : 5. Answer: 1 : 5 (water : milk)
- Three varieties of rice costing ₹20/kg, ₹25/kg and ₹30/kg are mixed in quantities 10 kg, 20 kg and 30 kg respectively. Find the average price of the mixture. Solution: With three components, use the weighted-average definition directly instead of the two-term alligation ratio. Total cost = (10 × 20) + (20 × 25) + (30 × 30) = 200 + 500 + 900 = ₹1,600 Total quantity = 10 + 20 + 30 = 60 kg Average price = 1,600 / 60 = ₹26.67/kg (80/3) Answer: ₹26.67/kg (80/3)
- A container has 40 litres of pure milk. 8 litres are withdrawn and replaced with water, and this process is repeated once more. Find the quantity of pure milk left after the two operations. Solution: Each operation removes a fraction x/V = 8/40 = 1/5 of whatever is currently in the container. After n operations, pure liquid left = $V \times (1 - x/V)^n = 40 \times (1 - 1/5)^2 = 40 \times (4/5)^2 = 40 \times 16/25 = 25.6$ litres Answer: 25.6 litres
- Practice: In what ratio should rice worth ₹50/kg be mixed with rice worth ₹70/kg so that the mixture is worth ₹55/kg? (Answer: 3 : 1 (₹50/kg rice : ₹70/kg rice). (70–55):(55–50) = 15:5.)
- Practice: A shopkeeper mixes 15 kg of sugar costing ₹30/kg with 25 kg of sugar costing ₹42/kg. Find the average price of the mixture. (Answer: ₹37.50/kg. Total cost = 450 + 1,050 = 1,500; total qty = 40; 1,500/40.)
- Practice: A mixture of 45 litres contains milk and water in the ratio 4 : 1. How much water must be added to make the ratio 2 : 1? (Answer: 9 litres. Milk = 36 L fixed; 36/(9+x) = 2/1 gives x = 9.)
- Practice: In what ratio must wheat at ₹24/kg be mixed with wheat at ₹32/kg to get a mixture worth ₹28/kg? (Answer: 1 : 1. (32–28):(28–24) = 4:4.)
- Practice: Rice at ₹18/kg, ₹24/kg and ₹30/kg is mixed in quantities 5 kg, 10 kg and 15 kg. Find the average price of the mixture. (Answer: ₹26/kg. Total cost = 90+240+450 = 780; total qty = 30; 780/30.)
- Practice: A vessel has 50 litres of pure milk. 10 litres are withdrawn and replaced with water, and this is repeated once more. Find the milk left after both operations. (Answer: 32 litres. $50 \times (1 - 10/50)^2 = 50 \times 0.64$.)

- Practice: A vessel contains 40 litres of pure milk. 10 litres are withdrawn and replaced with water (a single operation). Find the ratio of milk to water in the resulting mixture. (Answer: 3 : 1. Milk left = 30 L, water = 10 L.)

> **Quick revision:** Alligation rule: to mix a cheaper item (price C) and a dearer item (price D) so the mixture's mean price is M, the required ratio is cheaper : dearer = (D − M) : (M − C). This only makes sense when M is strictly between C and D — if the target mean is outside that range, the mixture as described is impossible; This is just the weighted-average equation rearranged — you can always sanity-check an answer by computing the weighted average directly from the ratio you found and confirming it equals the target mean; The same rule works for anything with a "concentration" — milk-water mixtures, alloy compositions, even mixing two classes' average marks — not just prices; For "cost price vs selling price" alligation questions (like diluting milk with free water to make a profit), treat water's cost as ₹0 and work out the mixture's effective cost price from the profit percentage first, before applying the ratio formula; For mixing three or more components at once, the two-term alligation shortcut does not directly apply — fall back to the weighted-average definition: total value of all components ÷ total quantity; Repeated dilution ("remove and replace"): if a container holds volume V of a pure liquid, and each time x units are withdrawn and replaced with water, then after n such operations the pure liquid remaining = $V \times (1 - x/V)^n$. This is a geometric decay, not a linear one — don't just subtract nx

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Partnership

Partnership — Concepts and Applications

When a problem assigns profit according to invested capital and time, each partner's weight is the sum of capital × duration over that partner's investment intervals. Divide distributable profit in the ratio of those weights, using the same time unit throughout. Joining, leaving or changing capital changes the relevant intervals rather than the agreed sharing principle. A stated salary or commission is deducted before sharing only when the question specifies that order, and any salary recipient may also receive a share of the remaining profit. Distinguish commission on total profit from commission on profit after commission, because the second requires an equation.

When a partner withdraws partway through the year (or joins partway through), their weight only counts the months they were actually invested - a partner who invests ₹30,000 but leaves after 8 months contributes a weight of 30,000 × 8, not 30,000 × 12, even though the total partnership period might be 12 months.

A simple commission (a straightforward percentage of total profit, taken before the rest is shared) is computed directly: commission = rate × total profit. This is different from the more subtle "commission on profit after commission" case, where the commission amount itself depends on what's left after subtracting it - that requires setting up an equation, since the unknown commission appears on both sides.

Examples

- Asha invests ₹20000 and Bharat ₹30000 for the same 12 months. They share ₹6600 profit in proportion to capital × months. Find Asha's share in rupees. Worked solution: The equal durations cancel, leaving weights 2:3. Asha gets 2/5 of ₹6600, or ₹2640.
- Asha invests ₹20000 for the first 6 months and ₹40000 for the next 6 months. Bharat invests ₹20000 for all 12 months. Profit ₹13200 is shared by capital × months. Find Asha's share in rupees. Worked solution: Asha's weight is 20000 × 6 + 40000 × 6 = 360000. Bharat's is 240000, so their ratio is 3:2. Asha's share is ₹7920.
- Total profit is ₹12100. Asha receives a commission equal to 10% of the profit remaining after her commission is deducted. What is the commission in rupees? Worked solution: Let commission be c. The stated base gives c = (p − c)/10, so 11c = p. Hence c = 12100/11 = ₹1100.
- A invests ₹24000 for the full 12 months. B invests ₹30000 but withdraws after 8 months. Profit of ₹11000 is shared by capital × months. Find each partner's share. Worked solution: A's weight = 24000×12 = 288000. B's weight = 30000×8 = 240000. Ratio = 288000:240000 = 6:5. A gets 6/11 × 11000 = ₹6000, and B gets 5/11 × 11000 = ₹5000.
- A profit of ₹20000 is to be shared where the manager first takes a straightforward 5% commission on the total profit, and the remainder is split equally between two partners. Find the manager's commission and each partner's share.

Worked solution: Commission = 5% × 20000 = ₹1000 (computed directly, with no equation needed since it's based on the total, not the remainder). Remaining ₹19000 is split equally: ₹9500 each.

> **Quick revision:** Profit-sharing weight = Σ(capital × duration); deduct stated salary/commission first, then share the remaining profit by these weights. A withdrawing/joining partner's weight only counts their actual invested months. Commission on TOTAL profit: compute directly (rate × total). Commission on profit AFTER commission: set up an equation, since the unknown appears on both sides.

Partnership Solved Questions

Partnership questions extend Ratio and Proportion to shared business profit — the one idea to hold onto is that profit splits by capital × time, not capital alone, whenever partners invest for different durations or amounts change mid-year. This page covers that core rule, mid-year capital changes, late-joining partners and working-partner fees, with several fully worked examples. If all partners invest for the SAME length of time, profit is split simply in the ratio of their capital amounts. If partners invest for DIFFERENT durations, each partner’s share of the ratio is capital × time — this product is what determines the split, not the capital amount on its own. This can produce a non-obvious ratio: a smaller capital invested for much longer can earn an equal or even larger share than a bigger capital invested briefly. If a partner changes their invested amount partway through the period (adding or withdrawing money), split their time into separate stretches and compute capital × time for each stretch, then add them up to get that partner’s total "capital-months". If a partner joins later than the others, their time factor is only the months their money was actually invested, not the full period — always base each partner's time on when THEIR capital entered the business. For a "working partner" who manages the business and earns an extra fixed fee for that on top of their investment return: subtract the fee from the total profit first, then split only the REMAINDER in the capital × time ratio. Reverse problems (given the profit-sharing ratio, find the investment ratio) just run the same rule backwards: capital × time ratio = profit ratio, so when the time periods are equal, the capital ratio equals the profit ratio directly.

Examples

- A and B start a business with capitals of ₹8,000 and ₹12,000 respectively, investing for the same time period. At the end of the year, they earn a profit of ₹5,000. Find each partner’s share. Solution: Since the time period is the same for both, the ratio is simply the capital ratio: 8,000 : 12,000 = 2 : 3. A’s share = (2/5) × 5,000 = ₹2,000. B’s share = (3/5) × 5,000 = ₹3,000. Answer: A gets ₹2,000, B gets ₹3,000
- A invests ₹10,000 for 12 months and B invests ₹15,000 for 8 months in a business. At the end, they earn a profit of ₹6,200. Find each partner’s share. Solution: The capitals alone (10,000 vs 15,000) are NOT the ratio to use, since the time periods differ — use capital × time instead. A: 10,000 × 12 = 1,20,000. B: 15,000 × 8 = 1,20,000. The two products are actually equal, so the ratio is 1 : 1, even though the capitals themselves weren’t equal. Each partner gets half of ₹6,200 = ₹3,100. Answer: A gets ₹3,100, B gets ₹3,100
- A, B and C invest in a business in the ratio 2 : 3 : 5, for the same time period. If the total profit is ₹20,000, find C’s share. Solution: Sum of ratio terms = 2 + 3 + 5 = 10. C’s share = (5/10) × 20,000 = ₹10,000. Answer: ₹10,000
- A invests ₹40,000 and manages the business, while B invests ₹60,000 as a sleeping partner. A receives 10% of the total profit as a management fee, and the rest is split in the ratio of their capitals. If the total profit is ₹25,000, find B’s total share. Solution: Management fee (A only) = 10% of 25,000 = ₹2,500. Remaining profit to split = 25,000 – 2,500 = ₹22,500. Capital ratio A : B = 40,000 : 60,000 = 2 : 3. B’s share of the remainder = (3/5) × 22,500 = ₹13,500. (B gets no management fee, so this is B’s total share.) Answer: ₹13,500
- A starts a business with ₹20,000. After 4 months, he adds a further ₹10,000. B joins at the very start with ₹25,000 and keeps it invested for the whole 12-month year. Find the profit-sharing ratio. Solution: A's capital-months in two stretches: ₹20,000 for the first 4 months, then ₹30,000 for the remaining 8 months. A: (20,000 × 4) + (30,000 × 8) = 80,000 + 2,40,000 = 3,20,000. B: 25,000 × 12 = 3,00,000. Ratio A : B = 3,20,000 : 3,00,000 = 32 : 30 = 16 : 15. Answer: 16 : 15
- A invests ₹30,000 at the start of the year. B joins the business after 3 months with ₹45,000. Find the profit-sharing ratio for that year. Solution: A's money is invested for the full 12 months; B's money is invested only for the remaining 12 – 3 = 9 months. A: 30,000 × 12 = 3,60,000. B: 45,000 × 9 = 4,05,000. Ratio A : B = 3,60,000 : 4,05,000 = 8 : 9 (dividing both by 45,000). Answer: 8 : 9

- A and B invest in a business for the same time period. Their profits come out in the ratio 4 : 5. If A invested ₹8,000, find B's investment. Solution: Since the time period is the same for both, the capital ratio equals the profit ratio: A : B = 4 : 5. 8,000 / B = 4 / 5 B = 8,000 × 5 / 4 = ₹10,000 Answer: ₹10,000
- Practice: P and Q start a business with capitals of ₹15,000 and ₹25,000, investing for the same time period. At the end of the year, they earn a profit of ₹8,000. Find each partner's share. (Answer: P gets ₹3,000, Q gets ₹5,000. Capital ratio 3:5 on a sum of 8.)
- Practice: X invests ₹12,000 for 4 months and Y invests ₹8,000 for 9 months. If the profit is ₹5,000, find each partner's share. (Answer: X gets ₹2,000, Y gets ₹3,000. Capital-months: X=48,000, Y=72,000, ratio 2:3.)
- Practice: A, B and C invest in a business in the ratio 3 : 4 : 5, for the same time period. If the total profit is ₹36,000, find A's share. (Answer: ₹9,000. (3/12) × 36,000.)
- Practice: P starts a business with ₹15,000. After 6 months, he withdraws ₹5,000 (leaving ₹10,000 invested). Q joins at the start with ₹12,000 and stays the full year. Find the profit-sharing ratio for the year. (Answer: 25 : 24. P: (15,000×6)+ (10,000×6) = 1,50,000; Q: 12,000×12 = 1,44,000; divide both by 6,000.)
- Practice: X starts a business with ₹24,000. Y joins after 4 months with ₹36,000. Find the profit-sharing ratio for the year. (Answer: 1 : 1. X: 24,000×12 = 2,88,000; Y: 36,000×8 = 2,88,000.)
- Practice: Two partners invest for the same time period in the ratio 3 : 7. If the total profit is ₹40,000, find each partner's share. (Answer: ₹12,000 and ₹28,000. (3/10) and (7/10) of 40,000.)
- Practice: A and B invest for the same time period. Their profits are ₹18,000 and ₹24,000 respectively. If B invested ₹40,000, find A's investment. (Answer: ₹30,000. Profit ratio 18,000:24,000 = 3:4 = capital ratio; A = 40,000 × 3/4.)

> **Quick revision:** If all partners invest for the SAME length of time, profit is split simply in the ratio of their capital amounts; If partners invest for DIFFERENT durations, each partner's share of the ratio is capital × time — this product is what determines the split, not the capital amount on its own; This can produce a non-obvious ratio: a smaller capital invested for much longer can earn an equal or even larger share than a bigger capital invested briefly; If a partner changes their invested amount partway through the period (adding or withdrawing money), split their time into separate stretches and compute capital × time for each stretch, then add them up to get that partner's total "capital-months"; If a partner joins later than the others, their time factor is only the months their money was actually invested, not the full period — always base each partner's time on when THEIR capital entered the business; For a "working partner" who manages the business and earns an extra fixed fee for that on top of their investment return: subtract the fee from the total profit first, then split only the REMAINDER in the capital × time ratio; Reverse problems (given the profit-sharing ratio, find the investment ratio) just run the same rule backwards: capital × time ratio = profit ratio, so when the time periods are equal, the capital ratio equals the profit ratio directly

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Permutation and Combination

Permutation & Combination

The whole topic comes down to one question: does the order of arrangement matter? Get that right and the correct formula follows automatically. This page covers that core distinction, repeated-letter arrangements, the "at least" and "always together" variants exams favour, and several fully worked examples. Permutation counts ARRANGEMENTS, where order matters (ABC is different from BCA). Combination counts SELECTIONS, where order doesn't matter (choosing the group {A, B, C} is the same regardless of order). Factorial is the building block for both: $n! = n \times (n-1) \times (n-2) \times \dots \times 1$, and by convention $0! = 1$. Permutation of n items taken r at a time: ${}^{n}P_{r} = n! / (n-r)!$. Combination of n items taken r at a time: ${}^{n}C_{r} = n! / (r! \times (n-r)!)$ — a combination is a permutation divided by the r! ways to arrange the chosen r items, since order no longer matters. Watch the wording: "arrange", "order", "ranks", "different ways to stand in a line" signal permutation. "Choose", "select", "form a committee" (where only the group matters, not who does what within it) signal combination. If a word has repeated letters, arranging all of them is NOT simply n! — divide by the factorial of each letter's repeat count: arrangements = $n! / (p! \times q! \times \dots)$, where p, q, … are how many times each distinct letter repeats. Fundamental counting principle: for a task done in several independent steps, each with its own number of choices, multiply the choices together. This is what lets you build up passwords,

codes or registration numbers digit by digit, not just count subsets. "At least" conditions (e.g. "a committee with at least 2 women") are usually solved by splitting into cases and adding (exactly 2, exactly 3, …), or by subtracting the complement (total ways minus the ways with fewer than the minimum) — whichever needs fewer cases. "Two particular people/items must always be together" in a row: glue them into a single block, arrange the reduced set of units, then multiply by the number of ways to arrange the people WITHIN that block (2! if it's a pair).

Examples

- In how many ways can the letters of the word "MANGO" be arranged? Solution: MANGO has 5 distinct letters, and arranging them is a permutation of all 5. Number of arrangements = 5! = 5 × 4 × 3 × 2 × 1 = 120. Answer: 120
- In how many ways can 3 books be selected from a shelf of 8 different books? Solution: Selecting books (the group chosen, not an order among them) is a combination. ${}^{8}C_{3}$ = 8! / (3! × 5!) = (8 × 7 × 6) / (3 × 2 × 1) = 336 / 6 = 56. Answer: 56
- In how many ways can a President and a Secretary be chosen from a group of 6 people, assuming one person can't hold both positions? Solution: The two roles are distinct (President ≠ Secretary), so who gets which role matters — this is a permutation. ${}^{6}P_{2}$ = 6! / 4! = 6 × 5 = 30. Answer: 30
- A committee of 3 people is to be formed from 5 men and 4 women, with exactly 2 men and 1 woman. In how many ways can this be done? Solution: Choosing 2 men from 5: ${}^{5}C_{2}$ = 10. Choosing 1 woman from 4: ${}^{4}C_{1}$ = 4. These two choices are independent, so multiply them: 10 × 4 = 40. Answer: 40
- In how many ways can the letters of the word "BANANA" be arranged? Solution: BANANA has 6 letters in total: B occurs once, A occurs 3 times, N occurs 2 times. Arrangements = 6! / (3! × 2!) = 720 / (6 × 2) = 720 / 12 = 60 Answer: 60
- A password consists of 2 letters (A–Z) followed by 3 digits (0–9), with repetition allowed in both parts. How many different passwords are possible? Solution: This is the fundamental counting principle applied step by step, not a selection of a subset. Letters: 26 choices for each of 2 positions = 26^{2} = 676. Digits: 10 choices for each of 3 positions = 10^{3} = 1,000. Total passwords = 676 × 1,000 = 6,76,000. Answer: 6,76,000
- A committee of 4 is to be formed from 6 men and 4 women, with at least 2 women. In how many ways can this be done? Solution: Split into cases by the exact number of women: 2, 3, or 4. Exactly 2 women, 2 men: ${}^{4}C_{2} \times {}^{6}C_{2}$ = 6 × 15 = 90. Exactly 3 women, 1 man: ${}^{4}C_{3} \times {}^{6}C_{1}$ = 4 × 6 = 24. Exactly 4 women, 0 men: ${}^{4}C_{4} \times {}^{6}C_{0}$ = 1 × 1 = 1. Total = 90 + 24 + 1 = 115. Answer: 115
- In how many ways can 5 people be seated in a row such that 2 particular people always sit together? Solution: Glue the 2 particular people into one block. This leaves 4 units to arrange: the block plus the other 3 people. These 4 units can be arranged in 4! = 24 ways. Within the block, the 2 people can switch places in 2! = 2 ways. Total = 24 × 2 = 48. Answer: 48
- Practice: In how many ways can the letters of the word "CHAIR" be arranged? (Answer: 120. All 5 letters distinct: 5!.)
- Practice: In how many ways can 4 students be selected from a class of 10 for a quiz team? (Answer: 210. ${}^{10}C_{4}$.)
- Practice: In how many ways can a committee of 2 men and 2 women be formed from 6 men and 5 women? (Answer: 150. ${}^{6}C_{2} \times {}^{5}C_{2}$ = 15 × 10.)
- Practice: In how many ways can the letters of the word "LEVEL" be arranged? (Answer: 30. L and E each repeat twice: 5!/(2!×2!) = 120/4.)
- Practice: A code consists of 3 digits (0–9), with repetition allowed. How many different codes are possible? (Answer: 1,000. 10^{3}, by the fundamental counting principle.)
- Practice: A committee of 3 is to be formed from 5 men and 3 women, with at least 1 woman. Find the number of ways. (Answer: 46. Total ${}^{8}C_{3}$ = 56; all-men (no woman) = ${}^{5}C_{3}$ = 10; 56 − 10.)
- Practice: In how many ways can 4 people be seated in a row such that 2 particular people always sit together? (Answer: 12. Glue the pair into 1 block (3 units total): 3! × 2! = 6 × 2.)

Quick revision: Permutation counts ARRANGEMENTS, where order matters (ABC is different from BCA). Combination counts SELECTIONS, where order doesn't matter (choosing the group {A, B, C} is the same regardless of order); Factorial is the building block for both: n! = n × (n–1) × (n–2) × … × 1, and by convention 0! = 1; Permutation of n items taken r at a time: ${}^{n}P_{r}$ = n! / (n–r)!. Combination of n items taken r at a time: ${}^{n}C_{r}$ = n! / (r! × (n–r)!) — a combination is a permutation divided by the r! ways to arrange the chosen r items, since order no longer matters; Watch the wording: "arrange", "order", "ranks", "different ways to stand in a line" signal permutation. "Choose", "select", "form a committee" (where only the group matters, not who does what within it) signal combination; If a word has repeated letters, arranging all of them is NOT simply n! — divide by the factorial of each letter's repeat count: arrangements = n! / (p! × q! × …), where p, q, … are how many times each distinct letter repeats; Fundamental counting principle: for a task done in several independent steps, each with its own number of choices, multiply the choices together. This is what lets you build up passwords, codes or registration numbers digit by digit, not just count subsets; "At least" conditions (e.g. "a committee with at least 2 women") are usually solved by splitting into cases and adding (exactly 2, exactly 3, …), or by subtracting the complement (total ways minus the ways with fewer than the minimum) — whichever needs fewer cases; "Two particular people/items must always be together" in a row: glue them into a single block, arrange the reduced set of units, then multiply by the number of ways to arrange the people WITHIN that block (2! if it's a pair)

Circular Permutations Solved Questions

Arranging people around a circle is different from arranging them in a row — rotating everyone one seat over gives the "same" arrangement, so one fixed reference point gets removed from the count. This page covers that core rule, when it does NOT apply (numbered/fixed seats), choosing a group before seating it, and alternating-gender seating, with several fully worked examples. n distinct people around a circular table: (n–1)! arrangements, not n! — fixing one person's seat removes the rotational duplicates (rotating everyone by one seat is still counted as the same arrangement). For a necklace or bracelet (where flipping it over gives the same arrangement, not just rotating it), divide by 2 as well: (n–1)!/2. When two specific people must sit together, glue them into a single block first. Arrange the resulting (reduced count) of entities in a circle using (count–1)!, then multiply by 2 for the two ways the glued pair can face each other. For "must NOT sit together," find the total circular arrangements, subtract the "sit together" count — this is faster and less error-prone than trying to count the "not together" cases directly. The (n–1)! rule assumes only RELATIVE position matters (an ordinary round table with unmarked seats). If the seats themselves are distinguishable — numbered chairs, or fixed compass directions — then rotating the whole group DOES create a genuinely new arrangement, and the count is the ordinary n!, just like arranging people in a row. To choose r people out of n and then seat that group around a circular table, first choose the group with ${}^{n}C_{r}$, then arrange it circularly with (r–1)!: total = ${}^{n}C_{r}$ × (r–1)!.

Examples

- In how many ways can 5 people be seated around a circular table? Solution: Circular arrangements of n people = (n–1)!. = (5–1)! = 4! = 24. Answer: 24
- In how many ways can 6 distinct beads be arranged to form a necklace? Solution: A necklace can be flipped over, so divide the circular count by 2: (n–1)!/2. = (6–1)!/2 = 120/2 = 60. Answer: 60
- In how many ways can 4 people be seated around a circular table if two particular people must always sit together? Solution: Glue the two people into one block: this leaves 3 entities (the block + 2 other people) to arrange circularly. Circular arrangements of 3 entities = (3–1)! = 2. The glued pair can sit in 2 internal orders (either person on the left), so total = 2 × 2 = 4. Answer: 4
- In how many ways can 7 people be seated around a circular table if two particular people must never sit together? Solution: Total circular arrangements of 7 people = (7–1)! = 720. Arrangements with the two together: glue them into a block (6 entities), circular arrangements = (6–1)! = 120, × 2 internal orders = 240. Never together = total – together = 720 – 240 = 480. Answer: 480
- In how many ways can 5 people be seated around a circular table that has 5 numbered seats (each seat is a distinct, labelled position)? Solution: Since the seats themselves are distinguishable, rotating the whole group produces a genuinely different arrangement (person X is now in seat 1 instead of seat 2, say). So this is NOT the usual circular case — it's equivalent to arranging 5 people in 5 distinct positions: 5! = 120. Answer: 120

- In how many ways can a committee of 4 people be chosen from 7 people and then seated around an ordinary (unmarked) circular table? Solution: First choose the 4 people: ${}^{7}C_{4} = 35$. Then arrange that group of 4 circularly: (4–1)! = 6. Total = 35 × 6 = 210. Answer: 210
- In how many ways can 4 men and 4 women be seated around a circular table so that men and women alternate? Solution: Fix one man's seat to remove rotational duplicates. The remaining 3 men can be arranged in the other 3 "male" seats in 3! ways. The 4 women occupy the 4 seats between the men, in 4! ways. Total = 3! × 4! = 6 × 24 = 144. Answer: 144
- Practice: In how many ways can 6 people be seated around a circular table? (Answer: 120)
- Practice: In how many ways can 5 distinct beads be arranged to form a necklace? (Answer: 12)
- Practice: In how many ways can 5 people be seated around a circular table if two particular people must always sit together? (Answer: 12)
- Practice: In how many ways can 6 people be seated around a circular table that has 6 numbered seats? (Answer: 720 (= 6!, since the seats are distinguishable).)
- Practice: In how many ways can a committee of 3 people be chosen from 6 people and then seated around an ordinary circular table? (Answer: 40. ${}^{6}C_{3} \times (3–1)! = 20 \times 2$.)
- Practice: In how many ways can 3 men and 3 women be seated around a circular table so that men and women alternate? (Answer: 12. (3–1)! × 3! = 2 × 6.)
- Practice: In how many ways can 8 people be seated around a circular table if two particular people must never sit together? (Answer: 3,600. Total (8–1)! = 5,040; together = (7–1)!×2 = 1,440; 5,040 – 1,440.)

> **Quick revision:** n distinct people around a circular table: (n–1)! arrangements, not n! — fixing one person's seat removes the rotational duplicates (rotating everyone by one seat is still counted as the same arrangement); For a necklace or bracelet (where flipping it over gives the same arrangement, not just rotating it), divide by 2 as well: (n–1)!/2; When two specific people must sit together, glue them into a single block first. Arrange the resulting (reduced count) of entities in a circle using (count–1)!, then multiply by 2 for the two ways the glued pair can face each other; For "must NOT sit together," find the total circular arrangements, subtract the "sit together" count — this is faster and less error-prone than trying to count the "not together" cases directly; The (n–1)! rule assumes only RELATIVE position matters (an ordinary round table with unmarked seats). If the seats themselves are distinguishable — numbered chairs, or fixed compass directions — then rotating the whole group DOES create a genuinely new arrangement, and the count is the ordinary n!, just like arranging people in a row; To choose r people out of n and then seat that group around a circular table, first choose the group with ${}^{n}C_{r}$, then arrange it circularly with (r–1)!: total = ${}^{n}C_{r} \times (r–1)!$

Problems on Ages

Problems on Ages: one variable, carried through

Assign a variable to one unknown age (usually the one appearing in the simplest ratio or relationship), then express every other age in the problem in terms of that same variable. "A is twice as old as B" means A = 2B - translate sentences in the same subject-first order they're written, not reversed. For "after N years" or "N years ago" conditions, add or subtract N from EVERY person's age in the problem, including the one already defined in terms of x - forgetting to adjust one person's age is the most common mistake here. When a ratio is given directly (like "ages are in the ratio 3 : 5"), use a single multiplier - 3x and 5x - rather than two separate unknowns, turning the problem into one variable solvable with just one more equation.

A powerful shortcut worth internalizing: the DIFFERENCE between two people's ages never changes over time. If a father is 30 years older than his son today, he was 30 years older ten years ago and will still be 30 years older twenty years from now. Recognizing this invariant often shortcuts problems that otherwise look like they need two variables - once the age difference is known, a single future or past ratio condition pins down both actual ages immediately.

When a problem links three people in a chain - "A is twice as old as B, and B is three times as old as C" - assign the variable to the person at the END of the chain (C, the one with the fewest multipliers attached) and express everyone

else going backward through the chain (B = 3x, A = 2×3x = 6x). This keeps every age as a clean multiple of a single x, exactly as the ratio case does.

Some problems give a future condition instead of a past one for a sum-based setup: "the sum of two present ages is S; after N years, one will be M times the other." The method is identical to the "N years ago" sum case - express one age as x, the other as (S – x), add N to both for the future condition, and solve the single resulting equation. Occasionally a problem combines both a past AND a future ratio condition on the same two people - this still reduces to one variable per person (two unknowns, two equations), solved the standard simultaneous-equations way rather than needing a ratio multiplier trick.

Examples

- A father is 3 times as old as his son; after 12 years he will be twice as old: 3x+12 = 2(x+12) gives x=12, so the son is 12 and the father 36.
- A and B's present ages are in the ratio 4:5; 8 years ago the ratio was 3:4: (4x-8)/(5x-8) = 3/4 gives x=8, so A is 32 and B is 40.
- A mother and daughter's ages sum to 40; 5 years ago the mother was 4 times the daughter's age: (35-x) = 4(x-5) gives x=11, so the daughter is 11 and the mother is 29.
- A father is 45 and his son is 15 today, a difference of 30 years. Ten years ago they were 35 and 5 (still 30 apart); twenty years from now they'll be 65 and 35 (still 30 apart) - the age gap between two people never changes.
- A is twice as old as B, and B is three times as old as C. If their ages sum to 50: let C = x, so B = 3x and A = 6x; then 6x+3x+x = 10x = 50 gives x = 5, so C is 5, B is 15, and A is 30.
- The sum of a mother and daughter's present ages is 50. After 5 years, the mother will be 3 times the daughter's age then: letting the daughter's present age be x, (50-x+5) = 3(x+5) gives x = 10, so the daughter is 10 and the mother is 40 - check: in 5 years, 45 = 3×15.
- 5 years ago, A's age was 3 times B's age; after 5 more years from now, A's age will be 2 times B's age. Solving the two equations A–5=3(B–5) and A+5=2(B+5) simultaneously gives B=15 and A=35.

> **Quick revision:** Assign x to one age, express all others in terms of x. 'After/ago N years' shifts EVERY age by N. A given ratio becomes one multiplier (3x, 5x), not two separate unknowns. The age DIFFERENCE between two people is constant over time - a powerful shortcut. Three-person chains: assign x to the person with the fewest multipliers. Two time conditions (past AND future) on the same pair need simultaneous equations.

Problems on Ages Solved Questions

Problems on Ages are really just algebra in disguise — assign a variable to one age, express every other age in terms of it, and the given relationship becomes a solvable equation. This page covers that approach, the age-difference sanity check, two-condition problems with two unknowns, and several fully worked examples. Assign a variable to one unknown age (usually the one appearing in the simplest ratio or relationship), then express every other age in the problem in terms of that same variable. "A is twice as old as B" means A = 2B — translate sentences in the same subject-first order they're written, not reversed. For "after N years" or "N years ago" conditions, add or subtract N from EVERY person's age in the problem, including the one already defined in terms of x — forgetting to adjust one person's age is the most common mistake here. When a ratio is given directly (like "ages are in the ratio 3 : 5"), use a single multiplier — 3x and 5x — rather than two separate unknowns. This turns the problem into one variable, solvable with just one more equation from the rest of the problem. Useful check: the DIFFERENCE between any two people's ages never changes over time. If you compute it once from the present ages, it must still hold true at every other time mentioned in the question — a quick way to catch an arithmetic slip. When a problem gives two separate conditions at two different times with genuinely two unknowns (not a simple ratio to start with), set up two equations in the same two variables and solve them together by substitution or elimination. After solving, re-check against EVERY condition in the question, not just the last one you used to solve for x — a value that satisfies one condition but not another means an earlier step went wrong.

Examples

- A father is 3 times as old as his son. After 12 years, the father will be twice as old as his son. Find their present ages. Solution: Let the son's present age = x, so the father's present age = 3x. After 12 years: father = 3x + 12, son = x + 12. Given: 3x + 12 = 2(x + 12) → 3x + 12 = 2x + 24 → x = 12. Son = 12, Father = 36. Check: after 12 years, father = 48, son = 24, and 48 = 2 × 24. Answer: Son = 12 years, Father = 36 years
- The present ages of A and B are in the ratio 4 : 5. Eight years ago, their ages were in the ratio 3 : 4. Find their present ages. Solution: Let A = 4x and B = 5x. Eight years ago: A – 8 = 4x – 8, B – 8 = 5x – 8. Given: (4x – 8) / (5x – 8) = 3 / 4. Cross-multiply: 4(4x – 8) = 3(5x – 8) → 16x – 32 = 15x – 24 → x = 8. A = 4 × 8 = 32, B = 5 × 8 = 40. Check: 8 years ago, A = 24, B = 32, and 24 : 32 = 3 : 4. Answer: A = 32 years, B = 40 years
- Five years ago, Rohan's age was three times his daughter's age. Ten years from now, Rohan's age will be twice his daughter's age. Find their present ages. Solution: Let the daughter's present age = x and Rohan's present age = y. Five years ago: y – 5 = 3(x – 5) → y = 3x – 10 … (1) Ten years from now: y + 10 = 2(x + 10) → y = 2x + 10 … (2) Setting (1) = (2): 3x – 10 = 2x + 10 → x = 20. Then y = 2(20) + 10 = 50. Answer: Daughter = 20 years, Rohan = 50 years
- The sum of the present ages of a mother and her daughter is 40 years. Five years ago, the mother's age was 4 times the daughter's age. Find their present ages. Solution: Let the daughter's present age = x, so the mother's present age = 40 – x. Five years ago: daughter = x – 5, mother = 35 – x. Given: 35 – x = 4(x – 5) → 35 – x = 4x – 20 → 55 = 5x → x = 11. Daughter = 11, Mother = 40 – 11 = 29. Check: 5 years ago, daughter = 6, mother = 24, and 24 = 4 × 6. Answer: Daughter = 11 years, Mother = 29 years
- A grandfather is 4 times as old as his grandson. In 8 years, the grandfather will be 3 times as old as the grandson will be then. Find their present ages. Solution: Let the grandson's present age = x, so the grandfather's present age = 4x. In 8 years: 4x + 8 = 3(x + 8) → 4x + 8 = 3x + 24 → x = 16. Grandson = 16, Grandfather = 64. Check: in 8 years, grandson = 24, grandfather = 72, and 72 = 3 × 24. Answer: Grandson = 16 years, Grandfather = 64 years
- Five years ago, the ratio of A's and B's ages was 3 : 5. Ten years from now, the ratio will be 3 : 4. Find their present ages. Solution: Let present ages be A and B. Five years ago: (A–5)/(B–5) = 3/5 → 5A – 25 = 3B – 15 → 5A – 3B = 10 … (1) Ten years from now: (A+10)/(B+10) = 3/4 → 4A + 40 = 3B + 30 → 4A – 3B = –10 … (2) Subtract (2) from (1): A = 20. Substitute into (2): 4(20) – 3B = –10 → 80 – 3B = –10 → B = 30. Check: 5 years ago, 15 : 25 = 3 : 5. 10 years from now, 30 : 40 = 3 : 4. Both match. Answer: A = 20 years, B = 30 years
- The difference between the ages of a father and his son is 24 years. In 4 years, the father will be 4 times as old as the son will be then. Find their present ages. Solution: Let the son's present age = x, so the father's present age = x + 24 (the age difference stays 24 years at every point in time). In 4 years: (x + 24) + 4 = 4(x + 4) → x + 28 = 4x + 16 → 3x = 12 → x = 4. Son = 4, Father = 28. Check the difference: 28 – 4 = 24. Check the given condition: in 4 years, father = 32, son = 8, and 32 = 4 × 8. Answer: Son = 4 years, Father = 28 years
- Practice: A mother is 4 times as old as her son. After 20 years, she will be twice as old as her son. Find their present ages. (Answer: Son = 10 years, Mother = 40 years)
- Practice: The present ages of P and Q are in the ratio 5 : 7. Six years ago, their ages were in the ratio 2 : 3. Find their present ages. (Answer: P = 30 years, Q = 42 years)
- Practice: Six years ago, a man was 4 times as old as his son. Six years from now, he will be twice as old as his son. Find their present ages. (Answer: Son = 12 years, Man = 30 years)
- Practice: A mother is 5 times as old as her daughter. In 6 years, she will be 3 times as old as the daughter will be then. Find their present ages. (Answer: Daughter = 6 years, Mother = 30 years. 5x+6 = 3(x+6) gives x = 6.)
- Practice: Five years ago, the ratio of X's and Y's ages was 1 : 2. Ten years from now, the ratio will be 2 : 3. Find their present ages. (Answer: X = 20 years, Y = 35 years. Set up (X–5)/(Y–5)=1/2 and (X+10)/(Y+10)=2/3, then solve together.)
- Practice: The difference between the ages of two friends is 15 years. Eight years ago, the elder was twice as old as the younger. Find their present ages. (Answer: Younger = 23 years, Elder = 38 years. Elder = younger+15; 8 years ago, (younger+15–8) = 2(younger–8).)
- Practice: The sum of the present ages of a man and his wife is 72 years. Four years ago, the ratio of their ages was 5 : 3. Find their present ages. (Answer: Man = 44 years, Wife = 28 years. (M–4)/(W–4)=5/3 with M+W=72.)

Quick revision: Assign a variable to one unknown age (usually the one appearing in the simplest ratio or relationship), then express every other age in the problem in terms of that same variable; "A is twice as old as B" means A = 2B — translate sentences in the same subject-first order they're written, not reversed; For "after N years" or "N years ago" conditions, add or subtract N from EVERY person's age in the problem, including the one already defined in terms of x — forgetting to adjust one person's age is the most common mistake here; When a ratio is given directly (like "ages are in the ratio 3 : 5"), use a single multiplier — 3x and 5x — rather than two separate unknowns. This turns the problem into one variable, solvable with just one more equation from the rest of the problem; Useful check: the DIFFERENCE between any two people's ages never changes over time. If you compute it once from the present ages, it must still hold true at every other time mentioned in the question — a quick way to catch an arithmetic slip; When a problem gives two separate conditions at two different times with genuinely two unknowns (not a simple ratio to start with), set up two equations in the same two variables and solve them together by substitution or elimination; After solving, re-check against EVERY condition in the question, not just the last one you used to solve for x — a value that satisfies one condition but not another means an earlier step went wrong

Free practice questions: parikshasaathi.com/q/problems-on-ages/

Profit, Loss and Discount

Profit, Loss and Discount — Concepts and Applications

Cost price includes all costs explicitly assigned to acquiring or preparing an item, while profit or loss equals the difference between selling price and cost price. Profit and loss percentages use cost price as their base, but a discount percentage uses marked price. Successive discounts multiply the retained-price factors, so they cannot generally be added. To recover cost or marked price, divide the final selling price by the relevant multiplier. For incorrect-weight transactions, compare revenue for the billed quantity with the actual cost of the quantity delivered, using the stated purchase cost and billing rule.

A dishonest dealer who claims to sell at cost price but uses a false (lighter) weight still makes a real profit: if a dealer bills for 1000 g but actually hands over only 900 g (while charging the price of 1000 g), the profit percentage is found just like a normal profit question, with "cost" being the 900 g actually given away and "selling price" being the 1000 g worth of money collected for it.

Given the final selling price AFTER a discount, the marked price is recovered by dividing (not subtracting): if a 10% discount was applied, the selling price is 90% of the marked price, so marked price = selling price ÷ 0.90, never selling price × 1.10 (that over-corrects, since 1.10 would undo an addition, not a percentage-of multiplication).

Examples

- An item costs ₹440 and sells for ₹488.4. Find the profit percentage on cost price. Worked solution: Profit = 488.4 − 440 = 48.4. Divide by the cost ₹440 and multiply by 100 to obtain 11%. Check: 11% of ₹440 is ₹48.4, exactly the selling-price excess.
- The marked price is ₹1600. Successive discounts of 10% and 20% are applied, the second to the already discounted price. Find the final price in rupees. Worked solution: After 10% off the price is ₹1440. Retaining 80% of that gives ₹1152; the combined retained factor is 0.9 × 0.8 = 0.72. Check: the effective discount is 28%, so subtracting 28% of ₹1600 gives the same price.
- An item costing ₹550 is marked 50% above cost and then sold at a 20% discount on that marked price. Find the profit percentage on cost. Worked solution: Marked price = 550 × 1.50 = ₹825. The selling price is 825 × 0.80 = ₹660. Profit divided by cost is (110/550) × 100 = 20%. Check the overall multiplier: 1.50 × 0.80 = 1.20, so the sale is 20% above cost.
- A dealer claims to sell at cost price but uses a weight of 900 g instead of 1000 g for every "kilogram" billed. Find his real profit percentage. Worked solution: He gives away only 900 g (his real cost) while charging for 1000 g (his effective selling price). Profit % = (1000−900)/900 × 100 = 100/9 ≈ 11.11%.
- After a 10% discount, an item sells for ₹720. Find its marked price. Worked solution: Selling price = 0.90 × marked price, so marked price = 720 ÷ 0.90 = ₹800 — dividing, not subtracting or adding 10% back.

- An item costing ₹500 is sold for ₹450. Find the loss percentage. Worked solution: Loss = 500 − 450 = 50. Loss % = (50/500) × 100 = 10%.
- A shopkeeper marks goods 40% above cost and allows a discount of 25% on the marked price. Find the overall profit or loss percentage. Worked solution: Overall multiplier = 1.40 × 0.75 = 1.05, meaning the final selling price is 105% of cost — a profit of 5%.

> **Quick revision:** Profit/loss% uses cost price; discount% uses marked price; selling price = marked price × retained discount factors (multiply, don't add, successive discounts). Include all stated costs. False-weight dealers: profit% = (claimed−actual)/actual × 100. Recovering marked price from a discounted selling price: divide by the retained fraction, never add the discount % back.

Profit and Loss Solved Questions

Profit and Loss builds directly on percentages, and the two are often tested together — a marked-price-and-discount question is really a percentage question wearing a different costume. This page covers the core formulas, the false-weight and markup-discount variants that exams favour, several fully worked examples, and practice questions with explained answers. Profit = Selling Price (SP) − Cost Price (CP), when SP > CP. Loss = CP − SP, when CP > SP. Profit% or Loss% is always calculated on the Cost Price unless a question explicitly says otherwise: (Profit or Loss / CP) × 100. A "profit margin on sales" (used in real business contexts) is calculated on SP instead — the two percentages are not the same number for the same rupee profit, so always check which base a question intends. Discount is calculated on the Marked Price (MP), not the cost price: SP = MP × (1 − Discount%/100). Mixing this up with CP is the most common mistake. When an item is marked up by a% and then discounted by b%, the net effect on profit is the same successive-change formula as percentages: net% = a + b + (ab/100). This also means a markup and a discount of the same size never cancel out — the result is always a net loss (see the worked example). False-weight or false-measure questions ("claims to sell at cost price but uses an understated weight") are profit questions in disguise: the dealer's gain% = (error / actual quantity given) × 100, where error = claimed quantity − actual quantity. A useful identity: if two items are each sold at the same selling price, one at a gain of x% and the other at a loss of x%, the overall result is always a loss of (x/10)% squared, i.e. (x^2/100)% — never a break-even, even though +x% and −x% look like they should cancel. Successive discounts (e.g. "20% off, then an extra 10% off") combine the same way as successive percentage changes, not by simply adding the discount percentages — see the dedicated Successive Discount lesson for more on this. Checks: a discount must bring SP below MP; a profit must bring SP above CP; profit% and loss% on the same CP are not interchangeable with profit% and loss% on different CPs without first converting to actual rupee amounts.

Examples

- A shopkeeper buys a table for ₹1,200 and sells it for ₹1,500. Find his profit percentage. Solution: Profit = 1,500 − 1,200 = 300 Profit% = (300 / 1,200) × 100 = 25% Answer: 25%
- A man sells a watch for ₹1,140 at a loss of 5%. Find the cost price. Solution: SP = CP × (1 − 5/100) = CP × 0.95 1,140 = CP × 0.95 CP = 1,140 / 0.95 = 1,200 Answer: ₹1,200
- An article marked at ₹2,000 is sold after a discount of 15%. Find the selling price. Solution: SP = MP × (1 − 15/100) = 2,000 × 0.85 = 1,700 Answer: ₹1,700
- A trader marks his goods 40% above cost price, then allows a discount of 10% on the marked price. Find his profit percentage. Solution: Let CP = 100. Marked Price = 100 × 1.40 = 140 SP = 140 × (1 − 10/100) = 140 × 0.90 = 126 Profit = 126 − 100 = 26, so Profit% = 26% Check with the successive-change formula: a=+40, b=−10 → net% = 40 − 10 + (40×−10)/100 = 30 − 4 = 26%. Answer: 26%
- A dishonest dealer claims to sell rice at cost price but uses a weight of 800 g instead of 1 kg. Find his gain percentage. Solution: He charges the cost-price rate for 1,000 g but actually hands over only 800 g, so his real cost is only for 800 g. Let the price per gram equal his cost per gram, c. Revenue collected = 1,000c (he bills for 1 kg). Actual cost = 800c (what he really gave). Gain = 1,000c − 800c = 200c. Gain% on his actual cost = (200c / 800c) × 100 = 25% Answer: 25%
- A trader marks an item 25% above its cost price but still ends up making a profit of only 12.5% after a discount. Find the discount percentage. Solution: Let CP = 100, so MP = 125 and (since profit is 12.5%) SP = 112.5. Discount amount

= MP − SP = 125 − 112.5 = 12.5 Discount% = (12.5 / 125) × 100 = 10% Answer: 10%

- Two items are each sold for ₹1,200. On one, a shopkeeper gains 20%; on the other, he loses 20%. Find his overall loss or gain percentage on this pair. Solution: CP of the first item = SP / 1.20 = 1,200 / 1.20 = 1,000. CP of the second item = SP / 0.80 = 1,200 / 0.80 = 1,500. Total CP = 1,000 + 1,500 = 2,500. Total SP = 1,200 + 1,200 = 2,400. Loss = 2,500 − 2,400 = 100. Loss% = (100 / 2,500) × 100 = 4%. Check against the identity: equal gain and loss of x% = 20% on equal selling prices always gives a loss of (x/10)% squared = $(20/10)^2$ = 4%. Matches. Answer: 4% loss (not a break-even, even though the percentages look equal and opposite)
- Practice: A man buys an item for ₹500 and sells it for ₹575. Find his profit percentage. (Answer: 15%. Profit = 75; 75/500 × 100 = 15%.)
- Practice: A bicycle is sold at a loss of 12% for ₹2,640. Find its cost price. (Answer: ₹3,000. CP × 0.88 = 2,640, so CP = 2,640/0.88.)
- Practice: Goods marked at ₹800 are sold at a discount of 25%. Find the selling price. (Answer: ₹600. SP = 800 × 0.75.)
- Practice: A dealer professes to sell at cost price but uses a weight of 900 g for 1 kg. Find his gain percentage. (Answer: 11.1% (1/9 × 100%). Gain% = (1,000 − 900)/900 × 100.)
- Practice: An item is marked up 40% above cost and then discounted by 20%. Find the net profit percentage. (Answer: 12%. Using net% = a + b + ab/100 = 40 − 20 + (40×−20)/100 = 20 − 8 = 12%.)
- Practice: Two articles are each sold for the same price: one at a 10% gain, the other at a 10% loss. Find the overall loss percentage on the pair. (Answer: 1%. Using $(x/10)^2$ with x = 10: $(10/10)^2$ = 1%.)
- Practice: A trader marks goods 20% above cost price and still makes a 4% profit after a discount. Find the discount percentage. (Answer: 13⅓%. MP = 120, SP = 104 (4% profit on CP 100); discount = 16 on MP 120, which is 16/120 × 100.)

> **Quick revision:** Profit = Selling Price (SP) − Cost Price (CP), when SP > CP. Loss = CP − SP, when CP > SP; Profit% or Loss% is always calculated on the Cost Price unless a question explicitly says otherwise: (Profit or Loss / CP) × 100. A "profit margin on sales" (used in real business contexts) is calculated on SP instead — the two percentages are not the same number for the same rupee profit, so always check which base a question intends; Discount is calculated on the Marked Price (MP), not the cost price: SP = MP × (1 − Discount%/100). Mixing this up with CP is the most common mistake; When an item is marked up by a% and then discounted by b%, the net effect on profit is the same successive-change formula as percentages: net% = a + b + (ab/100). This also means a markup and a discount of the same size never cancel out — the result is always a net loss (see the worked example); False-weight or false-measure questions ("claims to sell at cost price but uses an understated weight") are profit questions in disguise: the dealer's gain% = (error / actual quantity given) × 100, where error = claimed quantity − actual quantity; A useful identity: if two items are each sold at the same selling price, one at a gain of x% and the other at a loss of x%, the overall result is always a loss of (x/10)% squared, i.e. $(x^2/100)$% — never a break-even, even though +x% and −x% look like they should cancel; Successive discounts (e.g. "20% off, then an extra 10% off") combine the same way as successive percentage changes, not by simply adding the discount percentages — see the dedicated Successive Discount lesson for more on this; Checks: a discount must bring SP below MP; a profit must bring SP above CP; profit% and loss% on the same CP are not interchangeable with profit% and loss% on different CPs without first converting to actual rupee amounts

Successive Discount Solved Questions

When two or more discounts are applied one after another, they do NOT simply add up — each discount is calculated on the price that's already been reduced by the one before it, so successive discounts are always worth less to the shopper than their sum suggests. This page covers that core idea, combining three discounts two at a time, and reverse problems (given the final price, find one of the discounts), with several fully worked examples. Apply each discount one at a time, in order, to whatever price is left after the previous discount — never add the percentages together and apply them once. The single discount equivalent to two successive discounts d1% and d2% is d1 + d2 − (d1 × d2)/100 — always a little LESS than the simple sum d1 + d2, because the second discount is applied to an already-reduced price. Successive discounts almost always leave the buyer paying MORE than a single discount equal to their sum would — "25% and 10% off" is a smaller saving than a flat "35% off," even though 25 + 10 = 35. Two successive 50% discounts do NOT make an item free — the second 50% is off the

already-halved price, so the combined effective discount is 75%, not 100%. For THREE successive discounts, combine them two at a time using the same formula repeatedly: first combine d1 and d2 into one equivalent discount, then combine that result with d3 — this is easier than tracking three percentages in your head at once. Reverse problems (given the final price, or the overall equivalent discount, find one individual discount) just run the multiplication backwards: divide the overall multiplier by the known discount's multiplier to isolate the unknown one.

Examples

- An item with MRP ₹2000 is sold after two successive discounts of 20% and 10%. Find the final selling price. Solution: After the first discount: ₹2000 × (100 − 20)/100 = ₹1600. After the second discount: ₹1600 × (100 − 10)/100 = ₹1440. Answer: ₹1440
- Using the same item (MRP ₹2000, successive discounts of 20% and 10%), find the single discount percentage that would produce the same final price. Solution: Single equivalent discount = d1 + d2 − (d1 × d2)/100 = 20 + 10 − (20 × 10)/100 = 30 − 2 = 28%. Check: ₹2000 × (100 − 28)/100 = ₹1440, which matches the final price from successive discounts. Answer: 28%
- An item with MRP ₹4000 is sold after three successive discounts of 10%, 20% and 25%. Find the final selling price. Solution: After 10%: ₹4000 × 0.90 = ₹3600. After 20%: ₹3600 × 0.80 = ₹2880. After 25%: ₹2880 × 0.75 = ₹2160. Answer: ₹2160
- A shopkeeper offers two successive discounts of 25% and 10% on an item with MRP ₹4000, instead of a single flat discount of 30%. Which option is cheaper for the customer, and by how much? Solution: Successive discounts: ₹4000 × 0.75 × 0.90 = ₹2700. Single 30% discount: ₹4000 × 0.70 = ₹2800. The successive discounts give a lower price (₹2700 vs ₹2800) — cheaper for the customer by ₹100. Answer: Successive discounts, by ₹100
- Find the single discount equivalent to three successive discounts of 10%, 20% and 25%, by combining two at a time. Solution: Combine 10% and 20% first: 10 + 20 − (10×20)/100 = 30 − 2 = 28%. Now combine that 28% with the remaining 25%: 28 + 25 − (28×25)/100 = 53 − 7 = 46%. Check using multipliers: 0.90 × 0.80 × 0.75 = 0.54, which corresponds to a 46% discount (1 − 0.54 = 0.46). Matches. Answer: 46%
- An item with MRP ₹5,000 is sold after two successive discounts, the first being 20%, for a final price of ₹3,400. Find the second discount percentage. Solution: After the first discount: 5,000 × 0.80 = ₹4,000. Let the second discount be d2: 4,000 × (1 − d2/100) = 3,400, so (1 − d2/100) = 3,400/4,000 = 0.85. d2 = 15%. Answer: 15%
- Two successive discounts are applied to an item, the first being 10%. If the overall equivalent single discount is 28%, find the second discount. Solution: Overall multiplier = 1 − 0.28 = 0.72. First discount's multiplier = 1 − 0.10 = 0.90. Second discount's multiplier = 0.72 / 0.90 = 0.80, so the second discount = 20%. Answer: 20%
- Practice: An item with MRP ₹1500 is sold after two successive discounts of 10% and 20%. Find the final selling price. (Answer: ₹1080)
- Practice: Find the single discount percentage equivalent to two successive discounts of 15% each. (Answer: 27.75%)
- Practice: An item with MRP ₹1000 is sold after two successive discounts of 50% each. Find the final selling price (it is NOT free). (Answer: ₹250)
- Practice: Find the single discount equivalent to three successive discounts of 10%, 10% and 10%, by combining two at a time. (Answer: 27.1%. Combine first two: 19%; combine 19% with 10%: 19+10−1.9 = 27.1%. Check: $0.9^3 = 0.729$.)
- Practice: An item with MRP ₹3,000 is sold after two successive discounts, the first being 25%, for a final price of ₹1,800. Find the second discount. (Answer: 20%. After first discount: ₹2,250; 2,250×(1−d2/100)=1,800 gives d2=20.)
- Practice: Two successive discounts are applied, the first being 20%. If the overall equivalent discount is 28%, find the second discount. (Answer: 10%. Overall multiplier 0.72 ÷ first multiplier 0.8 = 0.9.)
- Practice: An item with MRP ₹6,000 is sold after successive discounts of 15% and 20%. Find the final selling price. (Answer: ₹4,080. 6,000 × 0.85 × 0.8.)

> **Quick revision:** Apply each discount one at a time, in order, to whatever price is left after the previous discount — never add the percentages together and apply them once; The single discount equivalent to two successive discounts d1% and d2% is d1 + d2 − (d1 × d2)/100 — always a little LESS than the simple sum d1 + d2, because the second discount is applied to an already-reduced price; Successive discounts almost always leave the buyer paying MORE than a single discount equal to their sum would — "25% and 10% off" is a smaller saving than a flat "35% off," even though 25 + 10 = 35; Two successive 50% discounts do NOT make an item free — the second 50% is off the already-halved price, so the combined effective discount is 75%, not 100%; For THREE successive discounts, combine them two at a time using the same formula repeatedly: first combine d1 and d2 into one equivalent discount, then combine that result with d3 — this is easier than tracking three percentages in your head at once; Reverse problems (given the final price, or the overall equivalent discount, find one individual discount) just run the multiplication backwards: divide the overall multiplier by the known discount's multiplier to isolate the unknown one

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Simplification and Roots

BODMAS, Fractions, Decimals, Roots and Indices

BODMAS means evaluate brackets first, then powers and roots, then multiplication and division from left to right, and finally addition and subtraction from left to right.

Add or subtract fractions using a common denominator, multiply numerators and denominators for products, divide by multiplying by the reciprocal of a nonzero divisor, and convert a mixed number a + b/c to (ac + b)/c.

For decimals, align decimal points when adding or subtracting, count the total decimal places when multiplying, and scale both numbers equally when dividing; a recurring decimal can be converted to a fraction by shifting the decimal point and subtracting to cancel identical repeating tails.

Compare fractions with positive denominators using a common denominator or cross-multiplication, so a/b < c/d exactly when ad < bc.

A square or cube means a number multiplied by itself two or three times, the principal square root is always non-negative, and the real cube root has the same sign as the original number.

For a positive base a and integer exponents, a^m × a^n = a^(m+n), a^m ÷ a^n = a^(m−n), (a^m)^n = a^(mn), a^0 = 1, and a^(−n) = 1/a^n.

Examples

- BODMAS with fractions and decimals: 0.75 + (2/3 ÷ 4/9) × 0.2 = 3/4 + (2/3 × 9/4) × 1/5 = 3/4 + 3/10 = 21/20 = 1.05; division and multiplication are completed before the final addition.
- Recurring decimals and comparison: if x = 0.1(6), where 6 repeats forever, then 100x = 16.(6) and 10x = 1.(6); subtraction gives 90x = 15, so x = 1/6; to compare 1/6 with 2/11, compare 1 × 11 = 11 with 2 × 6 = 12, proving 1/6 < 2/11.
- Roots and indices: √(144/225) = 12/15 = 4/5, ∛(−216) = −6, and 2^3 × 2^(−5) = 2^(−2) = 1/4; checking by squaring 4/5, cubing −6, and multiplying 8 × 1/32 verifies each result.

> **Quick revision:** Brackets → powers/roots → × and ÷ left to right → + and − left to right; fraction division uses reciprocals; recurring tails cancel by subtraction; compare ad and bc; principal √ is non-negative; add/subtract/multiply exponents for product/quotient/power of a power.

Simplification (BODMAS) Solved Questions

Simplification questions are a speed test more than a difficulty test — the only real skill is applying the BODMAS order correctly instead of solving strictly left to right. This page covers the order of operations, the vinculum (bar) rule and mixed fraction-decimal expressions, with several fully worked examples. BODMAS gives the order of operations: Brackets, Of, Division, Multiplication, Addition, Subtraction — solve innermost brackets first, not left to right across the whole expression. "Of" means multiply, but by the standard teaching convention for these exams, every "of" in

the expression is resolved before any separate division or multiplication is carried out — this is the step that trips people up when they skip straight to left-to-right. Division and multiplication rank EQUALLY with each other and are done left to right in the order they appear — it's not "always divide before multiplying". The same is true for addition and subtraction. Work brackets from the innermost outward: solve ( ) first, then { }, then [ ], substituting each result back in before moving to the next layer out. A vinculum (a bar drawn over part of an expression) has the highest priority of all — even higher than brackets. Simplify everything under the bar first, as its own self-contained BODMAS calculation, before touching anything outside it. When an expression mixes fractions and decimals, convert everything to one form (usually fractions, since they combine exactly) before calculating — combining 0.6 and 3/5 without converting one of them is a common source of arithmetic slips. Check your work by re-simplifying just the innermost piece on its own and confirming it matches what you substituted back in — this catches most "wrong order" mistakes before they compound through the rest of the expression.

Examples

- Simplify: 15 + 6 ÷ 3 × 2 − 4 Solution: Division and multiplication first, left to right: 6 ÷ 3 = 2, then 2 × 2 = 4. Now addition and subtraction, left to right: 15 + 4 − 4 = 15. Answer: 15
- Simplify: 5 of 8 ÷ 4 + 6 × 2 Solution: "Of" first: 5 of 8 = 40. Division and multiplication next: 40 ÷ 4 = 10, and 6 × 2 = 12. Addition last: 10 + 12 = 22. Answer: 22
- Simplify: 100 − [20 + {10 − (6 − 2)}] Solution: Innermost brackets first: (6 − 2) = 4. Next layer: {10 − 4} = 6. Next layer: [20 + 6] = 26. Finally: 100 − 26 = 74. Answer: 74
- Simplify: (3/4) of 60 + 25% of 80 − 15 Solution: "Of" terms first: (3/4) of 60 = 45, and 25% of 80 = 20. Addition and subtraction, left to right: 45 + 20 − 15 = 50. Answer: 50
- Simplify: 2/5 of 100 − (3/4 of 20 − 5) Solution: Innermost bracket first: 3/4 of 20 = 15, so the bracket becomes (15 − 5) = 10. Now the "of" outside the bracket: 2/5 of 100 = 40. Finally: 40 − 10 = 30. Answer: 30
- Simplify: 30 − bar(10 ÷ 2 + 3) × 2, where bar(...) denotes a vinculum — the expression under it is treated as a single term and simplified first, using its own internal BODMAS. Solution: Inside the vinculum, ordinary BODMAS still applies: 10 ÷ 2 = 5, then 5 + 3 = 8. So bar(10 ÷ 2 + 3) = 8, treated as one number. The rest of the expression now reads: 30 − 8 × 2. Multiplication before subtraction: 8 × 2 = 16, then 30 − 16 = 14. Answer: 14
- Simplify: 1.2 × 5/6 + 1.8 Solution: Convert the decimal to a fraction so everything is in one form: 1.2 = 6/5. (6/5) × (5/6) = 1 (the 5s and 6s cancel exactly — this is why converting first avoids rounding). 1 + 1.8 = 2.8. Answer: 2.8
- Practice: Simplify: 20 − 4 × 3 + 6 ÷ 2 (Answer: 11. Multiplication/division first: 4×3=12, 6÷2=3; then 20−12+3.)
- Practice: Simplify: 2 of 15 ÷ 5 − 3 × 2 (Answer: 0. "Of" first: 2 of 15 = 30; then 30÷5=6 and 3×2=6; 6−6.)
- Practice: Simplify: 50 − [15 + {20 − (8 + 4)}] (Answer: 27. Innermost: 8+4=12; {20−12}=8; [15+8]=23; 50−23.)
- Practice: Simplify: 36 ÷ 6 × 2 + 5 − 3 of 2 (Answer: 11. "Of" first: 3 of 2 = 6; then 36÷6=6, 6×2=12; 12+5−6.)
- Practice: Simplify: 30 − bar(10 ÷ 2 + 3) × 2, with bar(...) a vinculum. (Answer: 14. Inside the bar: 10÷2+3 = 8; then 30 − 8×2 = 30−16.)
- Practice: Simplify: 1.2 × 5/6 + 1.8 (Answer: 2.8. 1.2 = 6/5; (6/5)×(5/6) = 1; 1+1.8.)
- Practice: Simplify: 2/5 of 100 − (3/4 of 20 − 5) (Answer: 30. Bracket first: 3/4 of 20 = 15, so (15−5)=10; then 2/5 of 100 = 40; 40−10.)

Quick revision: BODMAS gives the order of operations: Brackets, Of, Division, Multiplication, Addition, Subtraction — solve innermost brackets first, not left to right across the whole expression; "Of" means multiply, but by the standard teaching convention for these exams, every "of" in the expression is resolved before any separate division or multiplication is carried out — this is the step that trips people up when they skip straight to left-to-right; Division and multiplication rank EQUALLY with each other and are done left to right in the order they appear — it's not "always divide before multiplying". The same is true for addition and subtraction; Work brackets from the innermost outward: solve ( ) first, then { }, then [ ], substituting each result back in before moving to the next layer out; A vinculum (a bar drawn over part of an expression) has the highest priority of all — even higher than brackets. Simplify everything under the bar first, as its own self-contained BODMAS calculation, before touching anything outside it; When an expression mixes fractions and decimals, convert everything to one form (usually fractions, since they combine exactly) before calculating — combining 0.6 and 3/5 without converting one of them is a common source of arithmetic slips; Check your work by re-simplifying just the innermost piece on its own and confirming it matches what you substituted back in — this catches most "wrong order" mistakes before they compound through the rest of the expression

Approximation Solved Questions

Approximation questions aren't asking for the exact answer — they're asking for the answer closest to the options given, so rounding every number first is the technique, not a shortcut around it. This page covers the core rounding approach, squares/cubes/roots, and how rounding errors behave differently in multiplication versus addition, with several fully worked examples. Round every number in the expression to a convenient nearby value FIRST — nearest whole number for decimals, nearest multiple of 10/100 for percentages and large numbers, nearest perfect square under a square root. Then calculate normally using BODMAS with the rounded numbers — the goal is a value close to the exact answer, not the exact answer itself. Round consistently: if you round one number up, the overall error is usually small as long as you don't round several numbers in the same direction by a large margin. This is a pure speed technique for multiple-choice questions where the options are spread far enough apart that an approximate value clearly identifies the right one. For squares, cubes and roots, round to the nearest number whose square, cube or root is easy to compute by hand — e.g. $28.9^2 \approx 29^2 = 841$, or the cube root of 215.5 ≈ cube root of 216 (a perfect cube) = 6. In a PRODUCT or DIVISION of several rounded numbers, rounding more than one number in the SAME direction compounds the error (both errors multiply together); rounding some up and some down tends to partially cancel it out. In a SUM or DIFFERENCE of several terms, rounding errors simply add (or partially cancel if some terms round up and others round down) — this is generally a smaller risk than in multiplication, where the errors combine multiplicatively rather than just adding.

Examples

- 41.9% of 599.8 ≈ ? Solution: Round: 41.9% → 42%, and 599.8 → 600. 42% of 600 = (42/100) × 600 = 252. Answer: 252
- √255.8 × 4.02 ≈ ? Solution: Round: 255.8 → 256 (a perfect square), and 4.02 → 4. √256 = 16, so 16 × 4 = 64. Answer: 64
- 12.98 × 7.03 + 25.02 ≈ ? Solution: Round: 12.98 → 13, 7.03 → 7, and 25.02 → 25. 13 × 7 + 25 = 91 + 25 = 116. Answer: 116
- 3599 ÷ 59.8 ≈ ? Solution: Round: 3599 → 3600, and 59.8 → 60. 3600 ÷ 60 = 60. Answer: 60
- $28.9^2 \approx$? Solution: Round 28.9 to the nearest whole number: 29. $29^2 = 841$. Answer: 841
- Cube root of 215.5 ≈ ? Solution: 215.5 is very close to 216, which is a perfect cube ($6^3 = 216$). So the cube root of 215.5 ≈ the cube root of 216 = 6. Answer: 6
- 20.9 × 30.8 ≈ ? Solution: Round: 20.9 → 21, and 30.8 → 31. Both were rounded UP, so the approximation will come out a little higher than the exact value — fine for a multiple-choice question with options spread apart. 21 × 31 = 651 (the exact value, 20.9 × 30.8, works out to 643.72 — close enough to clearly identify the right option). Answer: ≈ 651
- Practice: 20.2% of 749.9 ≈ ? (Answer: 150)
- Practice: 15.92 × 4.08 ≈ ? (Answer: 64)
- Practice: 6399 ÷ 79.7 ≈ ? (Answer: 80)
- Practice: $19.1^2 \approx$? (Answer: 361. Round 19.1 to 19; 19^2.)

- Practice: Cube root of 63.5 ≈ ? (Answer: 4. 63.5 is close to 64 = 4^3.)
- Practice: 11.2 × 8.9 ≈ ? (Answer: 99. Round to 11 × 9.)
- Practice: 49.8 + 30.2 − 19.9 ≈ ? (Answer: 60. Round to 50 + 30 − 20.)

> **Quick revision:** Round every number in the expression to a convenient nearby value FIRST — nearest whole number for decimals, nearest multiple of 10/100 for percentages and large numbers, nearest perfect square under a square root; Then calculate normally using BODMAS with the rounded numbers — the goal is a value close to the exact answer, not the exact answer itself; Round consistently: if you round one number up, the overall error is usually small as long as you don't round several numbers in the same direction by a large margin; This is a pure speed technique for multiple-choice questions where the options are spread far enough apart that an approximate value clearly identifies the right one; For squares, cubes and roots, round to the nearest number whose square, cube or root is easy to compute by hand — e.g. $28.9^2 \approx 29^2 = 841$, or the cube root of 215.5 ≈ cube root of 216 (a perfect cube) = 6; In a PRODUCT or DIVISION of several rounded numbers, rounding more than one number in the SAME direction compounds the error (both errors multiply together); rounding some up and some down tends to partially cancel it out; In a SUM or DIFFERENCE of several terms, rounding errors simply add (or partially cancel if some terms round up and others round down) — this is generally a smaller risk than in multiplication, where the errors combine multiplicatively rather than just adding

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Stocks and Shares

Stocks and Shares Solved Questions

Three values matter for every share — Face Value (the printed value, dividend is always a percentage of this), Market Value (what you actually pay), and the dividend income itself — and most questions just connect these three through one investment. This page covers that connection, the "r% ₹N stock at M" quoting convention exams commonly use, and comparing which of two stocks is the better buy, with several fully worked examples. Face Value (FV) is the share's printed/nominal value (commonly ₹100 unless stated otherwise) — dividend is always calculated as a percentage of FV, never of the market price. Market Value (MV) is the price actually paid to buy the share. A share "at a premium of ₹X" means MV = FV + X; "at a discount of ₹X" means MV = FV − X. Number of shares bought = Investment ÷ MV. Annual income = Number of shares × Dividend per share, where dividend per share = (Dividend % × FV) / 100. Rate of return (yield) is the dividend as a percentage of what was actually paid: Yield% = (Dividend per share / MV) × 100 — this is what lets you compare returns across shares bought at different prices. Stock is often quoted differently in exam wording: "r% ₹N stock at M" means a holding with total FACE VALUE ₹N, paying r% dividend on that face value, currently trading at market price M (per ₹100 of face value). Cost to buy this holding = N × M/100. Annual income = r% of N (the face value) — not of the cost paid. To decide which of two stocks is the better investment, compare their yields (return per rupee actually invested): Yield% = (Dividend% × FV) / MV. The higher yield is the better buy, even if its advertised dividend percentage looks smaller.

Examples

- A man buys a ₹100 share at ₹120. If the dividend is 8%, find his income on an investment of ₹2,400. Solution: Number of shares = 2,400 / 120 = 20. Dividend per share = 8% of ₹100 = ₹8. Income = 20 × 8 = ₹160. Answer: ₹160
- A ₹100 share paying 10% dividend is bought at a premium of ₹25. Find the rate of return (yield) on the investment. Solution: Market value = 100 + 25 = ₹125. Dividend per share = 10% of ₹100 = ₹10. Yield = (10 / 125) × 100 = 8%. Answer: 8%
- A man invests ₹1,800 in buying ₹100 shares at a discount of 10%, and receives a dividend of 12%. Find his annual income. Solution: Market value = 100 − 10 = ₹90. Number of shares = 1,800 / 90 = 20. Dividend per share = 12% of ₹100 = ₹12. Income = 20 × 12 = ₹240. Answer: ₹240
- Find the market value of a ₹100 share paying 10% dividend, if the rate of return (yield) to the investor is 8%. Solution: Dividend per share = 10% of ₹100 = ₹10. Yield% = (Dividend / MV) × 100, so 8 = (10 / MV) × 100. MV = (10 / 8) × 100 = ₹125. Answer: ₹125

- Find the cost of ₹7,500, 8% stock at 96, and the annual income from it. Solution: Cost = Face Value × (Market price / 100) = 7,500 × 96/100 = ₹7,200. Annual income = 8% of the face value = 8% of 7,500 = ₹600. Answer: Cost = ₹7,200, Income = ₹600
- Which is the better investment: a 9% stock at 120, or an 11% stock at 143? Solution: Yield of first stock = (9/120) × 100 = 7.5%. Yield of second stock = (11/143) × 100 ≈ 7.69%. The second stock gives a slightly higher yield, so it is the better investment, even though its market price is higher. Answer: The 11% stock at 143 (yield ≈ 7.69% vs 7.5%)
- A man spends ₹9,600 to buy a 10% stock at 96. Find the face value of stock he purchased and his annual income. Solution: Face value = Cost × 100 / Market price = 9,600 × 100/96 = ₹10,000. Annual income = 10% of the face value = 10% of 10,000 = ₹1,000. Answer: Face value = ₹10,000, Income = ₹1,000
- Practice: A man buys a ₹100 share at ₹150. If the dividend is 9%, find his income on an investment of ₹3,000. (Answer: ₹180)
- Practice: A ₹100 share paying 12% dividend is bought at a premium of ₹20. Find the rate of return. (Answer: 10%)
- Practice: A man invests ₹1,600 in buying ₹100 shares at a discount of 20%, and receives a dividend of 15%. Find his annual income. (Answer: ₹300)
- Practice: Find the cost of ₹5,000, 10% stock at 110, and the annual income from it. (Answer: Cost = ₹5,500, Income = ₹500.)
- Practice: Which is the better investment: a 6% stock at 80, or a 9% stock at 135? (Answer: The 6% stock at 80 (yield 7.5% vs ≈6.67%).)
- Practice: A man spends ₹4,800 to buy a 12% stock at 96. Find the face value of stock purchased and his annual income. (Answer: Face value = ₹5,000, Income = ₹600.)
- Practice: A ₹100 share paying 15% dividend is bought at a premium of ₹50. Find the yield. (Answer: 10%. MV = 150; (15/150) × 100.)

> **Quick revision:** Face Value (FV) is the share's printed/nominal value (commonly ₹100 unless stated otherwise) — dividend is always calculated as a percentage of FV, never of the market price; Market Value (MV) is the price actually paid to buy the share. A share "at a premium of ₹X" means MV = FV + X; "at a discount of ₹X" means MV = FV – X; Number of shares bought = Investment ÷ MV. Annual income = Number of shares × Dividend per share, where dividend per share = (Dividend % × FV) / 100; Rate of return (yield) is the dividend as a percentage of what was actually paid: Yield% = (Dividend per share / MV) × 100 — this is what lets you compare returns across shares bought at different prices; Stock is often quoted differently in exam wording: "r% ₨N stock at M" means a holding with total FACE VALUE ₨N, paying r% dividend on that face value, currently trading at market price M (per ₨100 of face value). Cost to buy this holding = N × M/100. Annual income = r% of N (the face value) — not of the cost paid; To decide which of two stocks is the better investment, compare their yields (return per rupee actually invested): Yield% = (Dividend% × FV) / MV. The higher yield is the better buy, even if its advertised dividend percentage looks smaller

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Trigonometry

Trigonometry — Concepts and Applications

In a right triangle, relative to an acute angle θ, sinθ is opposite/hypotenuse, cosθ is adjacent/hypotenuse and tanθ is opposite/adjacent. Cosecθ, secθ and cotθ are the reciprocals of sinθ, cosθ and tanθ respectively. The identities $\sin^2\theta + \cos^2\theta = 1$, $\sec^2\theta - \tan^2\theta = 1$ and $\operatorname{cosec}^2\theta - \cot^2\theta = 1$ hold wherever the ratios are defined. Complementary angles satisfy $\sin(90° - \theta) = \cos\theta$ and $\tan(90° - \theta) = \cot\theta$, with corresponding reciprocal relationships. The sine values at 0°, 30°, 45°, 60° and 90° are 0, 1/2, $1/\sqrt{2}$, $\sqrt{3}/2$ and 1, and the corresponding cosine values occur in reverse order. Use the angle's stated range to choose signs and check nonzero denominators before simplifying expressions.

Once any two sides of a right triangle are known, every one of the six ratios can be found directly - there's no need to find the angle itself first. From a 3-4-5 triangle (opposite 3, adjacent 4, hypotenuse 5), all six ratios fall out immediately: sin=3/5, cos=4/5, tan=3/4, and their reciprocals cosec=5/3, sec=5/4, cot=4/3.

The complementary-angle identities are a useful shortcut when a problem mixes θ and (90°–θ): rather than treating them as two different unknown angles, recognize that sin and cos simply swap roles (and so do tan/cot, sec/cosec)

between an angle and its complement.

Examples

- In a right triangle, the side opposite acute angle θ is 6 cm and the hypotenuse is 10 cm. Find sinθ. Worked solution: Sine is opposite/hypotenuse. Hence sinθ = 6/10 = 3/5. Check: the remaining leg is 8, and $(3/5)^2 + (4/5)^2 = 1$.
- Evaluate $\sin^2(0°) + \cos^2(0°)$. Worked solution: At 0°, $\sin^2 = 0$ and $\cos^2 = 1$. Substituting gives 0 + 1 = 1.
- For an acute θ, sinθ = 4/5 and cosθ = 3/5. Evaluate sinθ/(1 + cosθ). Worked solution: The denominator is 1 + 3/5 = 8/5. Dividing 4/5 by this gives 0.5.
- In a 3-4-5 right triangle (opposite 3, adjacent 4, hypotenuse 5 relative to angle θ), find secθ, cosecθ and cotθ. Worked solution: secθ = hyp/adj = 5/4, cosecθ = hyp/opp = 5/3, cotθ = adj/opp = 4/3. Check: $\sec^2\theta - \tan^2\theta = (5/4)^2 - (3/4)^2 = 25/16 - 9/16 = 1$, confirming the identity.
- If θ has sinθ = 3/5 and cosθ = 4/5, find sin(90°−θ) and cos(90°−θ) without finding θ itself. Worked solution: By the complementary identities, sin(90°−θ) = cosθ = 4/5, and cos(90°−θ) = sinθ = 3/5 - the two ratios simply swap for the complementary angle.
- Verify $\sec^2 30° - \tan^2 30° = 1$ using the standard values. Worked solution: sec30° = 2/√3 and tan30° = 1/√3. Then $\sec^2 30° - \tan^2 30° = 4/3 - 1/3 = 3/3 = 1$, confirming the identity at a standard angle.

> **Quick revision:** sinθ=opp/hyp, cosθ=adj/hyp, tanθ=opp/adj; cosec, sec, cot are their reciprocals. $\sin^2\theta+\cos^2\theta=1$; $\sec^2\theta-\tan^2\theta=1$; $\text{cosec}^2\theta-\cot^2\theta=1$. Complementary angles swap sin↔cos, tan↔cot, sec↔cosec. Any two known sides of a right triangle give all six ratios directly, without needing the angle itself. Standard values at 0°,30°,45°,60°,90°: sin = 0, 1/2, 1/√2, √3/2, 1 (cos reversed).

Trigonometric Ratios & Identities

Unlike Height and Distance, which applies trigonometry to real-world triangles, this topic works with the ratios and identities directly — given one ratio, find another, or simplify an expression using a fixed identity. This page covers the core and derived identities, reciprocal and complementary-angle relationships, and edge values at 0°/90°, with several fully worked examples. The core identity: $\sin^2\theta + \cos^2\theta = 1$, for any angle θ. Given one of sin θ or cos θ, this gives the other directly. Two derived identities, both built from the core one: $1 + \tan^2\theta = \sec^2\theta$, and $1 + \cot^2\theta = \text{cosec}^2\theta$. Standard angle values worth knowing by heart: sin 30° = cos 60° = 1/2; sin 60° = cos 30° = √3/2; sin 45° = cos 45° = 1/√2; tan 45° = 1; tan 30° = 1/√3; tan 60° = √3. For an acute angle, all ratios are positive, so taking the square root in an identity only ever needs the positive root — this simplifies working with $\sin^2\theta + \cos^2\theta = 1$ in practice. Reciprocal relationships: cosec θ = 1/sin θ, sec θ = 1/cos θ, cot θ = 1/tan θ — convert between a ratio and its reciprocal this way rather than re-deriving a triangle each time. Complementary angle relationships: sin(90° − θ) = cos θ, cos(90° − θ) = sin θ, and tan(90° − θ) = cot θ — useful for simplifying expressions that mix an angle with its complement. Edge values worth knowing: sin 0° = 0, cos 0° = 1, sin 90° = 1, cos 90° = 0. tan 90° and cot 0° are undefined (they involve dividing by zero), which is why some identities only hold for angles strictly between 0° and 90°.

Examples

- If sin θ = 3/5 and θ is acute, find cos θ. Solution: Using $\sin^2\theta + \cos^2\theta = 1$: $\cos^2\theta = 1 - (3/5)^2 = 1 - 9/25 = 16/25$. cos θ = √(16/25) = 4/5 (positive, since θ is acute). Answer: 4/5
- Evaluate: $2\sin^2 45° + 3\cos^2 45°$ Solution: sin 45° = cos 45° = 1/√2, so $\sin^2 45° = \cos^2 45° = 1/2$. 2(1/2) + 3(1/2) = 1 + 1.5 = 2.5. Answer: 2.5
- If tan θ = 3/4 and θ is acute, find sec θ. Solution: Using $1 + \tan^2\theta = \sec^2\theta$: $\sec^2\theta = 1 + (3/4)^2 = 1 + 9/16 = 25/16$. sec θ = √(25/16) = 5/4. Answer: 5/4
- Evaluate: (sin 30° + cos 60°) / (1 + tan 45°) Solution: sin 30° = 1/2 and cos 60° = 1/2, so the numerator = 1/2 + 1/2 = 1. tan 45° = 1, so the denominator = 1 + 1 = 2. Result = 1/2. Answer: 1/2
- If cos θ = 5/13 and θ is acute, find sec θ and cosec θ. Solution: cos θ = 5/13 corresponds to a 5-12-13 right triangle, so sin θ = 12/13. sec θ = 1/cos θ = 13/5. cosec θ = 1/sin θ = 13/12. Answer: sec θ = 13/5, cosec θ = 13/12
- Evaluate: sin 35° / cos 55° Solution: cos 55° = cos(90° − 35°) = sin 35°, using the complementary angle relationship. So the expression becomes sin 35° / sin 35° = 1. Answer: 1

- Simplify: $(1 - \cos^2\theta)(1 + \cot^2\theta)$ Solution: $1 - \cos^2\theta = \sin^2\theta$, from the core identity. $1 + \cot^2\theta = \text{cosec}^2\theta = 1/\sin^2\theta$, from the derived identity. Multiplying: $\sin^2\theta \times (1/\sin^2\theta) = 1$. Answer: 1
- Practice: If $\sin\theta = 5/13$ and θ is acute, find $\cos\theta$. (Answer: 12/13)
- Practice: If $\tan\theta = 5/12$ and θ is acute, find $\sec\theta$. (Answer: 13/12)
- Practice: Evaluate: $3\sin^2 60° + 2\cos^2 30°$ (Answer: 3.75)
- Practice: If $\sin\theta = 7/25$ and θ is acute, find $\text{cosec}\,\theta$ and $\cos\theta$. (Answer: $\text{cosec}\,\theta = 25/7$, $\cos\theta = 24/25$ (a 7-24-25 right triangle).)
- Practice: Evaluate: $\tan 20° / \cot 70°$ (Answer: 1. $\cot 70° = \cot(90°–20°) = \tan 20°$.)
- Practice: Simplify: $(1 + \tan^2\theta) \times \cos^2\theta$ (Answer: 1. $1+\tan^2\theta = \sec^2\theta = 1/\cos^2\theta$, so the product is 1.)
- Practice: Evaluate: $\sin 90° + \cos 0° - \sin 0°$ (Answer: 2. $1 + 1 - 0$.)

> **Quick revision:** The core identity: $\sin^2\theta + \cos^2\theta = 1$, for any angle θ. Given one of $\sin\theta$ or $\cos\theta$, this gives the other directly; Two derived identities, both built from the core one: $1 + \tan^2\theta = \sec^2\theta$, and $1 + \cot^2\theta = \text{cosec}^2\theta$; Standard angle values worth knowing by heart: $\sin 30° = \cos 60° = 1/2$; $\sin 60° = \cos 30° = \sqrt{3}/2$; $\sin 45° = \cos 45° = 1/\sqrt{2}$; $\tan 45° = 1$; $\tan 30° = 1/\sqrt{3}$; $\tan 60° = \sqrt{3}$; For an acute angle, all ratios are positive, so taking the square root in an identity only ever needs the positive root — this simplifies working with $\sin^2\theta + \cos^2\theta = 1$ in practice; Reciprocal relationships: $\text{cosec}\,\theta = 1/\sin\theta$, $\sec\theta = 1/\cos\theta$, $\cot\theta = 1/\tan\theta$ — convert between a ratio and its reciprocal this way rather than re-deriving a triangle each time; Complementary angle relationships: $\sin(90° - \theta) = \cos\theta$, $\cos(90° - \theta) = \sin\theta$, and $\tan(90° - \theta) = \cot\theta$ — useful for simplifying expressions that mix an angle with its complement; Edge values worth knowing: $\sin 0° = 0$, $\cos 0° = 1$, $\sin 90° = 1$, $\cos 90° = 0$. $\tan 90°$ and $\cot 0°$ are undefined (they involve dividing by zero), which is why some identities only hold for angles strictly between 0° and 90°

Free practice questions: parikshasaathi.com/q/trigonometry/

Revision sheet

Every lesson’s quick-revision box in one place, for the last read before the exam.

Average — Concepts and Applications	Average = total/count; weighted average = Σ(value × weight)/Σweights; use totals before combining, adding, removing or replacing values. Replacing one member: new value = old value ± (count × change in average). Consecutive/evenly-spaced numbers: average = (first+last)/2, which equals the middle term when the count is odd. Combining groups: add totals and counts, never average the averages directly unless the groups are equal size.
Average Solved Questions	Average = Sum of all values / Number of values; Reversed, this gives Sum = Average × Number of values — this reversal is the key to most average questions, which usually give you an average and ask for something about the sum (or vice versa); When a new value joins a group, work out the new total sum first (old sum + new value), then divide by the new count — never average the old average directly with the new value; Useful shortcut: if adding one new value to n existing values changes the average from A to A′, the new value itself = A′ + n(A′ − A); For a set of consecutive numbers (or any evenly spaced sequence), the average equals the middle value (odd count) or the average of the two middle values (even count) — equivalently, (first + last) / 2. This is faster than adding every term; Combining two groups: the combined average is the weighted average by group size, $(sum_1 + sum_2) / (count_1 + count_2)$ — not the simple average of the two group averages, unless the two groups happen to be the same size; Deviation method for quick mental averaging: pick a convenient round base near the numbers, find how much each number deviates from it (plus or minus), average those small deviations, then add the result back to the base; Sanity check: the average of a set must lie between its minimum and maximum value. If a computed average falls outside that range, there is a calculation error somewhere
Data Interpretation — Concepts and Applications	Totals add, differences subtract, ratios compare in the stated order, percentages divide by the specified base. Combined average = total quantity/total count (weighted when frequencies differ). Pie sector value = (sector angle/360°) × total. Ratio: simplify by the HCF. Percentage change = (new − old)/old × 100 - always divide by the OLD value unless told otherwise.
Data Interpretation Solved Questions	Always check the units and any scale given with the table or chart ("figures in thousands," "sales in ₹ lakhs") before calculating — misreading the scale is the single most common careless mistake in this topic; For "what percentage more/less" questions, calculate the percentage change relative to the ORIGINAL value (the thing being compared against), not the new value — the same base-of-comparison rule from Percentages, just inside a table; For multi-step questions, compute each individual value needed FIRST and write it down, rather than trying to combine several lookups in your head at once; A ratio or an average computed from table data works exactly like the standalone Ratio/Average topics — the table just supplies the raw numbers; When a table mixes two related series for the same categories (e.g. sales AND cost, or income AND expenditure), a "profit" or "difference" question needs row-by-row subtraction — never average or otherwise combine the two series directly; If a table gives a grand total and leaves one entry blank, find the missing value by subtracting the sum of the known entries from the given total; After answering, sanity-check the result against the direction the data is actually moving — if every year in a table shows rising sales, a computed "decrease" should make you go back and recheck a step

Pie Chart Solved Questions	Every pie chart totals 360° (a full circle) and 100% — each category's slice is a percentage of the whole, or equivalently a fraction of 360°; If a category's percentage is given directly, its value = (percentage/100) × total. If its angle (in degrees) is given instead, first convert: percentage = (angle/360) × 100; To combine two or more categories, add their percentages (or their degrees) first, then apply the total once — don't calculate each category's value separately and add those, since rounding can creep in; To find a category that isn't stated directly, subtract the sum of the given percentages (or degrees) from 100% (or 360°); To compare two categories within the SAME pie chart, you can skip converting to rupee values entirely and just compare their angles or percentages as a ratio — e.g. if Rent is 30% and Food is 25% of the same chart, Rent : Food = 30 : 25 = 6 : 5, regardless of what the total actually is; Comparing the SAME category across two DIFFERENT pie charts (such as two different years) by percentage alone can be misleading if the totals differ — a smaller percentage of a much larger total can still be a bigger actual amount than a larger percentage of a smaller total. Always convert both to actual values before comparing across charts
Bar Graph Solved Questions	Each bar's height (or length, for a horizontal bar graph) represents that category's value on the scale shown on the axis — read it carefully against the axis markings before calculating anything; A percentage change between two bars uses the same rule as any other percentage question: (new − old) / old × 100, with the EARLIER bar as the original value; For "which period had the highest/lowest change" questions, calculate every period-to-period change first and write them all down, rather than trying to eyeball which bars look most different; An average across several bars is just their sum divided by the count — the bar graph format doesn't change the formula, only how you read the input values; Some bar graphs show TWO or more bars per category (a "grouped" bar graph) — e.g. separate bars for Boys and Girls in each year. Keep the series separate throughout a calculation; only add or average across series if the question explicitly asks for a combined figure; Watch for a y-axis that doesn't start at zero (a "broken" or truncated axis) — bars can look dramatically different in height even when the underlying percentage difference between them is small. Always calculate from the actual labelled values on the axis, never from how tall the bars visually appear
Line Graph Solved Questions	Each point on the line represents a value at that year (or period) on the scale shown on the axis — read it against the axis markings, not by guessing from the slope; A percentage change between two points uses the same rule as any other percentage question: (new − old) / old × 100, with the EARLIER point as the original value; A steep rising segment means a big increase and a steep falling segment means a big decrease — but "steep" is only a rough guide; for "which period had the largest change" questions, calculate every segment's change first rather than eyeballing the slope; An average across several points is just their sum divided by the count, exactly like a bar graph or table — the format doesn't change the formula, only how you read the input values; Some line graphs plot TWO or more lines on the same axes (e.g. the sales of two different companies over the same years) — read each line separately against the shared scale. A "when did A overtake B" question just means finding the first plotted point where A's value exceeds B's; Never extrapolate a trend beyond the plotted points. A line graph only tells you what happened at the years actually shown — projecting the slope forward into an unshown year assumes the pattern continues exactly, which the chart itself does not confirm
Number System: Divisibility, Factors, Powers and Remainders	$N = dq + r$; factor count = $\prod$(exponent + 1); square/cube exponents are multiples of 2/3; factorial prime exponent = Σ floor(n/p^j); odd sum = n^2; even sum = n(n + 1); inclusive index count = last − first + 1.

Number System Solved Questions	Divisibility rules let you check whether a number is divisible by a small number without doing the division: by 2 if the last digit is even; by 3 if the digit sum is divisible by 3; by 4 if the last two digits form a number divisible by 4; by 5 if the last digit is 0 or 5; by 9 if the digit sum is divisible by 9; by 11 if the difference between the sum of digits in odd and even positions (counting from the right) is divisible by 11; When a number N is divided by a divisor D, $N = D \times Q + R$, where Q is the quotient and R is the remainder, with $0 \le R < D$. This relationship is the foundation of every remainder question; Remainders behave predictably under addition and multiplication: you can take the remainder of each piece first, rather than computing a huge expression in full — useful for "find the remainder of a large power" questions, where the remainders of successive powers usually repeat in a short cycle; A number is a perfect square if every exponent in its prime factorization is even; it's a perfect cube if every exponent is a multiple of 3; Useful summation formulas: sum of the first n natural numbers = $n(n+1)/2$; sum of the first n squares = $n(n+1)(2n+1)/6$; sum of the first n cubes = $[n(n+1)/2]^2$; To find the unit digit of a^n, find the (short) cycle of unit digits that powers of a repeat through, then use n mod (cycle length) to pick the right position — if that remainder comes out to 0, it points to the LAST entry of the cycle, not necessarily a's own unit digit; Number of trailing zeros in n! = how many times 5 divides into the numbers from 1 to n (count multiples of 5, then 25, then 125, … and add). Count only the 5s, not the 2s — a factorial always has more factors of 2 than of 5, so the 5s are the limiting factor; Number of factors of N, once you have its prime factorization $N = p_1^a \times p_2^b \times \ldots$, is $(a+1) \times (b+1) \times \ldots$ — each exponent contributes its own count of choices (0 through that exponent), multiplied together
Percentage Conversions, Changes and Applications	p% = p/100; share = part/whole × 100; change% = (new − old)/old × 100; new = old × (1 ± p/100); original = new ÷ multiplier; successive multipliers multiply; savings = income − expenditure.
Percentage Solved Questions	"Percent" just means "per hundred" — x% of a number N is $(x/100) \times N$; Percentage change (increase or decrease) is always calculated on the original value: (change / original value) × 100; For two successive percentage changes of a% and b% (either can be negative, for a decrease), the net change is $a + b + (ab/100)$ — not simply a + b. This is the single most common mistake in percentage questions; Before calculating, name the base: "A is 20% more than B" uses B as 100%. Reversing the comparison changes the base, so the reverse percentage is different; A percentage multiplier is $1 + r/100$ for an increase and $1 - r/100$ for a decrease. To recover an original value, divide the final value by the multiplier; do not subtract the same percentage from the final value; A change in a rate from 40% to 50% is 10 percentage points, but a 25% relative increase: $(50 - 40)/40 \times 100$. State which measure you are using; For a combined percentage, add the underlying quantities before dividing. Averaging percentages is valid only when their bases are equal; Useful checks: an increase followed by an equal percentage decrease leaves a loss; a value after a discount must be below its original value; a percentage of a total cannot exceed 100% unless the comparison allows it
Ratios, Proportions, Sharing and Ages	Ratio a:b compares in stated order; simplify to lowest terms. a:b=c:d implies ad=bc. Sharing a total T in ratio a:b gives shares $Ta/(a+b)$ and $Tb/(a+b)$. A difference matches the DIFFERENCE in ratio parts, not the full terms. Compound ratio of a:b and c:d = ac:bd. Mean proportional x between a,b: $x^2=ab$. Inverse sharing uses 1/a:1/b = b:a. Ages: express as kx, shift all ages by the same elapsed time for past/future ratios.

Ratio and Proportion Solved Questions	A ratio a : b compares two quantities of the same kind and has no units. Multiplying or dividing both terms by the same non-zero number gives an equivalent ratio; The parts of a ratio are not the actual quantities unless a total or one actual value is also given — a : b = 3 : 5 really means the two quantities are 3x and 5x for some common multiplier x, which must be found from the rest of the question; A proportion is two equal ratios: a : b = c : d, which means a × d = b × c ("product of extremes = product of means") — this cross-multiplication is how most proportion questions get solved; To split a total T in the ratio a : b : c, each part = (that term ÷ sum of terms) × T; To combine two ratios a : b and b : c that share a term into one three-term ratio a : b : c, scale each ratio so the shared term (b) matches in both, usually by finding their LCM; Two quantities are in direct proportion if their ratio stays constant (a/b = k); they are in inverse proportion if their product stays constant (a × b = k). Mixing these up is a common mistake — for a fixed distance, speed and time are inversely related, not directly, because a higher speed means less time, not more; To compare two ratios a : b and c : d, cross-multiply: a × d versus b × c. If a × d > b × c, then a : b > c : d; In a continued proportion a : b = b : c, the middle term b is the mean proportional between a and c, and b = √(a × c)
Mean, Third & Fourth Proportional	Fourth proportional to a, b, c: find x such that a : b = c : x. Cross-multiplying gives x = (b × c) / a; Third proportional to a, b: find x such that a : b = b : x — note b is repeated. Cross-multiplying gives x = b^2 / a; Mean proportional between a and b: find x such that a : x = x : b — x is repeated this time. Cross-multiplying gives x^2 = a × b, so x = √(ab); The three are easy to mix up because they all involve three given-or-implied numbers — write out the proportion statement explicitly first, with the unknown in its correct position, before cross-multiplying; All three describe a geometric progression (a sequence with a constant ratio between consecutive terms): the third proportional makes a, b, x into a GP; the mean proportional makes a, x, b into a GP. Recognising this gives you a fast sanity check — divide consecutive terms and confirm the ratio is the same each time; Reverse problems give you the proportional value itself and ask for one of the original three numbers — rearrange the same cross-multiplied equation to isolate whichever variable is now the unknown, rather than solving for the usual one; Check your answer by substituting it back into the original proportion as an actual ratio (e.g. 3:5 should equal 12:20 after simplifying) to confirm the two sides genuinely match
Simple and Compound Interest — Concepts and Applications	SI = Prt/100; CI amount = P(1 + r/100)^t for annual periods; half-yearly uses r/2 and 2t; changing rates multiply their period factors (never average them). For exactly 2 years: CI–SI = P×$(r/100)^2$. To find rate or time from SI: rearrange rate=(SI×100)/(Pt) or time=(SI×100)/(Pr).
Simple & Compound Interest	Simple Interest (SI) = (P × R × T) / 100, where P is principal, R is the annual rate, T is time in years. Amount = P + SI; Compound Interest (CI), compounded annually: Amount = P × $(1 + R/100)^T$. CI = Amount − P; For the same P, R and T (with T > 1), CI is always greater than SI — interest earns interest in later years. When T = 1 year, CI and SI are identical, because no compounding has happened yet; Shortcut for 2 years: CI − SI = P × $(R/100)^2$. Shortcut for 3 years: CI − SI = P × $(R/100)^2$ × (3 + R/100). Both follow directly from expanding $(1 + R/100)^T$, and both are only valid for the stated number of years; Compounding over 2 years is the same idea as applying the same percentage increase twice in a row (see the Percentages lesson): the net percentage growth over 2 years is 2R + R^2/100, which is why CI for 2 years is always more than 2 × (one year's SI); If the rate changes each year (e.g. 10% in year 1, then 20% in year 2), do not use a single formula — multiply the year-by-year multipliers: Amount = P × $(1 + R_1/100)$ × $(1 + R_2/100)$ × ; A common mistake is plugging CI numbers into the SI formula (or vice versa) because a question mentions "interest" without saying which kind — always check the words "simple" or "compound" before choosing a formula; For a time period with a fraction of a year (e.g. 2 years 6 months) compounded annually, the standard convention used in these exams is: compute CI normally for the whole years, then add simple interest for the fractional part on the amount reached after the whole years — not compound interest on the fraction itself, unless a question says otherwise

Half-Yearly & Quarterly Compound Interest	For half-yearly compounding: divide the annual rate by 2, and multiply the time (in years) by 2, before applying Amount = P × (1 + rate/100)^periods; For quarterly compounding: divide the annual rate by 4, and multiply the time (in years) by 4; The underlying formula never changes — only what counts as "the rate per period" and "the number of periods" does. Get those two numbers right first, then it's the same calculation as annual compounding; Compound Interest = Amount − Principal, exactly as with annual compounding — this last step is easy to forget after focusing on the rate/time adjustment; For the same principal, annual rate and time, more frequent compounding always earns slightly MORE interest: quarterly beats half-yearly, which beats annual — because interest starts earning interest on itself sooner when periods are shorter; Reverse problems (given the final amount, find the principal or the rate) work exactly like annual compounding, just with the already-adjusted per-period rate and number of periods: Principal = Amount / (1 + rate/100)^periods, or compare the amount factor to standard powers to recover the rate
Time and Work — Concepts and Applications	Work = rate × time; combined rates add, emptying rates subtract; account for each work interval separately; wages follow completed work when stated. For worker-count questions: M1×D1 = M2×D2 (men-days constant). Wage shares split in the exact ratio of individual rates when working the same duration.
Time and Work Solved Questions	If a person finishes a piece of work in n days, their work rate is 1/n of the work per day. Converting everyone to a rate like this is the key move in almost every question here; When people work together, ADD their individual daily rates to get the combined rate. Time to finish together = 1 ÷ (combined rate); Shortcut for two people, A in x days and B in y days, working together from the start: combined time = xy / (x + y). (This is just the rate-addition rule worked out algebraically — useful to recall directly.); "X% more efficient" means X% more work done per day, i.e., the rate is multiplied by (1 + X/100) — and since rate and time are inverses, the time is divided by the same factor, not multiplied; For the same total work, men and days trade off: $Men_1 \times Days_1 = Men_2 \times Days_2$ (add × Hours per day on both sides if daily working hours change too). For a different amount of work W, the general form is (M × D × H) / W = constant; When two people work on alternate days, work out how much gets done in one full cycle (e.g. one day each), see how many whole cycles fit before the work is close to finished, then handle the leftover day by day — do not assume the alternation divides evenly unless it actually does; When someone leaves partway through (or joins partway through), split the problem into stages: find the work done in the stage where everyone present was working together, subtract it from 1, then solve the remaining stage with whoever is left; Check: after working through a multi-stage problem, add up every stage's contribution — it must total exactly 1 (one full job). If it doesn't, a stage was computed incorrectly
Pipes and Cisterns Solved Questions	A pipe that FILLS a tank has a positive rate (fraction of the tank filled per hour); a pipe that EMPTIES it (an outlet) has a negative rate. Combine rates exactly like Time and Work, but outlet pipes subtract; If a filling pipe can fill a tank in x hours, its rate is 1/x. If an outlet can empty a full tank in y hours, its rate is −1/y; With multiple pipes open together, add up all the rates (filling positive, emptying negative) to get the net rate. Time to fill (or empty) = 1 ÷ |net rate|, and the sign tells you which direction the tank is actually moving; A pipe that's "twice as fast" as another has half the time — rate and time are inverses here just like in Time and Work; Reverse problems (given the combined time, find one pipe's own time) just invert the usual setup: rearrange 1/x ± 1/y = 1/(combined time) algebraically for the unknown rate, then take its reciprocal to get that pipe's own time; If one pipe starts before another joins in, or a pipe is closed partway through, split the problem into stages exactly like in Time and Work: find the fraction of the tank filled in each stage, subtract running totals from 1, and solve the final stage with whichever pipe(s) remain

Work and Wages Solved Questions	If everyone works for the SAME number of days, wages are split in the ratio of their individual one-day work rates (1/days each person alone would take) — faster workers get a bigger share; Find each person's one-day work rate first (1/days alone), simplify the ratio between them, and only then divide the total wage in that ratio; For three or more people, the rate of the one not given directly can be found by subtracting the known rates from the combined rate — exactly like Time and Work; A larger wage share does not always mean more DAYS worked — it means a HIGHER work rate (faster individual completion time), since everyone here works the same number of days together; The "split by rate ratio" rule above only applies when everyone works the SAME number of days. If someone works fewer days than another (joins late, leaves early, or the days worked are simply stated as different), wages must be split by actual WORK DONE — each person's rate × the number of days THEY actually worked — not by the raw one-day rates alone; To handle this, compute each person's actual fraction of the job completed (rate × their own days worked), confirm these fractions add up to the whole job (1), and use those fractions — not the bare rates — as the wage-split ratio
Time, Speed and Distance — Concepts and Applications	Distance = speed × time; 1 m/s = 18/5 km/h; average speed = total distance/total time; use relative speed (sum for opposite, difference for same direction) for meetings and crossings. Trains crossing a point use only their own length; crossing a platform/bridge or another train adds both lengths. Race head-start ratio = full distance : (distance − head start).
Time, Speed and Distance Solved Questions	Speed = Distance / Time. Rearranged: Distance = Speed × Time, and Time = Distance / Speed; To convert km/hr to m/s, multiply by 5/18. To convert m/s to km/hr, multiply by 18/5; When the SAME distance is covered at two different speeds a and b (for example, going and returning by the same route), the average speed for the whole journey is 2ab/(a + b), not the simple average (a + b)/2. Using (a + b)/2 here is the single most common mistake in this topic — it only gives the right answer when the TIME spent at each speed is equal, not the distance; For a journey with different distances at different speeds (the equal-distance shortcut does not apply), fall back to the definition: average speed = total distance / total time. This always works, even when the shortcut above does not; For two objects moving toward each other, relative speed is the sum of their speeds; the time to meet (or to cross, for trains) uses this combined speed. For two objects moving in the same direction, relative speed is the difference of their speeds; Meeting-point problems: if two bodies start at the same time from two points a distance D apart and move toward each other at speeds a and b, they meet after time D/(a + b), at a distance a × D/(a + b) from the first point; Stoppage-time problems: if a train's speed excluding stoppages is s_1 and its effective (average, including stoppages) speed is s_2, the time lost to stoppages per hour of travel = $(1 - s_2/s_1) \times 60$ minutes; Keep units consistent throughout a single calculation — mixing km with minutes, or m/s with hours, without converting first is a common source of wrong answers even when the method is right
Problems on Trains Solved Questions	Crossing a stationary point (a pole, a man standing still, a signal) means the train covers a distance equal to its OWN length — treat the point as having zero length; Crossing a platform (or bridge) means the train covers a distance equal to its own length PLUS the platform's length — the whole train must clear the whole platform; Two trains crossing each other: use relative speed (sum of speeds if opposite directions, difference if same direction), and the distance to cover is the SUM of both train lengths — the same rule as objects, just with two lengths now; If the train crosses a person who is also moving (walking, not standing still), use relative speed exactly as with two trains — add the speeds for opposite directions, subtract for the same direction — but the distance to cover is still just the train's own length, since a person has no length to add; "Meet, then continue" problems: if two trains start towards each other and, after crossing, take a and b hours respectively to reach the OTHER train's starting point, then (speed of train 1) / (speed of train 2) = $\sqrt{(b/a)}$; Always convert km/hr to m/s (× 5/18) before working with lengths in metres and time in seconds — mixing units is the most common mistake here

Races and Games Solved Questions	"A beats B by X metres" means: at the moment A finishes, B still has X metres left to run; "A gives B a start of X metres" means B only has to cover (total distance – X) to finish — B effectively starts X metres ahead; "A gives B a start of X seconds" means B starts running X seconds before A does — add X seconds to A's finishing time to get B's head-start-adjusted clock; For a "dead heat" (both finish together), set up the condition that both cover their required distance in exactly the same time, then solve for the unknown; A "distance start" and a "time start" convert into each other once you know the slower runner's speed: a start of T seconds is equivalent to a start of (T × that runner's speed) metres, and vice versa; For three runners, where you're told how much A beats B by and how much B beats C by (as a fraction or percentage of the race distance), combine the two speed RATIOS by multiplying them — don't just add the two "beats by" distances, since each one is measured against a different base
Algebra — Concepts and Applications	$(a\pm b)^2 = a^2\pm 2ab+b^2$; $(a+b)(a-b)=a^2-b^2$; cancel factors only when nonzero; rationalise with conjugates; $y=mx+c$ has slope m. Quadratics: factor into $(x-p)(x-q)=0$ or use the quadratic formula. Simultaneous equations: eliminate one variable by matching coefficients. Cube identities: $a^3\pm b^3 = (a\pm b)(a^2\mp ab+b^2)$.
Surds and Indices Solved Questions	For powers of the SAME base: $a^m \times a^n = a^{m+n}$ (add exponents when multiplying); $a^m \div a^n = a^{m-n}$ (subtract when dividing); $(a^m)^n = a^{mn}$ (multiply when raising a power to a power); Any nonzero number to the power 0 equals 1 ($a^0 = 1$). A negative exponent means "reciprocal": $a^{-n} = 1/a^n$; A surd is an irrational root that can't be simplified to a whole number, like $\sqrt{2}$ or $\sqrt{3}$ ($\sqrt{4} = 2$ is NOT a surd, since it simplifies exactly). To simplify one, pull out the largest perfect-square factor: $\sqrt{50} = \sqrt{(25 \times 2)} = 5\sqrt{2}$; To "rationalize" a fraction with a surd in the denominator, multiply top and bottom by that surd (or its conjugate, if the denominator has two terms), so the denominator becomes a whole number: $1/\sqrt{2} = \sqrt{2}/2$; Fractional exponents are roots: a^(1/n) = $\sqrt[n]{a}$ (the nth root of a), and a^(m/n) = ($\sqrt[n]{a}$)^m — apply the root and the power in either order, whichever is easier to compute by hand; To solve an equation where both sides are powers of the same base, equate just the exponents: if a^x = a^y (with a > 0, a ≠ 1), then x = y; Only "like" surds (the same number under the root) combine directly, by adding or subtracting their coefficients — $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$, but $\sqrt{2} + \sqrt{3}$ cannot be merged into a single surd term. To compare two different surds, square both sides to remove the root, then compare the squared values
Quadratic Equations Solved Questions	Solve equation I for all values of x, and equation II for all values of y — each quadratic usually gives two roots; Compare every x with every y. If x is bigger in every single combination, the answer is "x > y"; if x is always smaller, it's "x < y"; If some combinations give x bigger and others give x equal, the answer is "x ≥ y" (and symmetrically "x ≤ y") — not "x > y", since equality also occurs; If the combinations genuinely conflict — some give x > y, others x < y (or x = y) — no single relationship covers every case, so the answer is "cannot be determined"; When an equation doesn't factor neatly into integers, use the quadratic formula: for $ax^2 + bx + c = 0$, $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$. The discriminant $D = b^2 - 4ac$ tells you the nature of the roots before you even finish solving: D > 0 gives two distinct real roots, D = 0 gives one repeated real root, D < 0 gives no real roots at all; Shortcut without fully solving: sum of roots = –b/a, and product of roots = c/a. A negative product means the two roots have opposite signs (one positive, one negative) — worth checking early, since a negative root often breaks what would otherwise look like a clean "x > y" or "x < y" pattern; Common mistakes: dropping the ± when taking a square root in the quadratic formula, and sign slips while factoring — always re-expand a factored form mentally to confirm it reproduces the original equation

Simple Equations Solved Questions	Assign a variable to the unknown quantity the question actually asks for, and translate each sentence into an equation using that variable (and a second variable if there are two unknowns); Phrases translate directly: "sum of" → +, "difference of" → –, "X more than Y" → Y + X, "X less than Y" → Y – X, "twice/three times a number" → 2x/3x; With two unknowns, you need two independent equations — write both before trying to solve either one; Once solved, substitute your answer back into BOTH original sentences (not just the equation) to confirm it actually satisfies the word problem, not just the algebra; For two unknowns where the equations aren't simply "sum" and "difference" (e.g. mixed quantities at different rates), use elimination — scale one or both equations so a variable's coefficient matches, then subtract to cancel it — or substitution — solve one equation for one variable and plug it into the other; For "consecutive numbers" questions, express them all in terms of one variable with a fixed step: x, x+1, x+2, … for consecutive integers; x, x+2, x+4, … for consecutive even or odd numbers. Their given sum or relationship then becomes one simple equation
Algebraic Identities Solved Questions	The two core identities: $(a+b)^2 = a^2 + 2ab + b^2$, and $(a–b)^2 = a^2 – 2ab + b^2$. Rearranged, each one connects a^2+b^2 to $(a+b)^2$ or $(a–b)^2$ and ab — given any two of the three, find the third; Difference of squares: $a^2 – b^2 = (a+b)(a–b)$ — this is the shortcut behind multiplying two numbers that are equally far from a round number, like 97×103 = (100–3)(100+3); When a question gives a+b (or a–b) and ab, it's testing whether you know $a^2+b^2 = (a+b)^2 – 2ab$ (or $(a–b)^2 + 2ab$) — don't try to find a and b individually first; These identities work in both directions: use them to expand an expression, or to go from a^2+b^2 and a+b (or a–b) back to find ab; Sum and difference of cubes: $a^3+b^3 = (a+b)(a^2–ab+b^2)$, and $a^3–b^3 = (a–b)(a^2+ab+b^2)$ — use these to factor a cube expression, such as $x^3–8 = (x–2)(x^2+2x+4)$; A faster route to a^3+b^3 or $a^3–b^3$ when you already know a+b (or a–b) and ab: $a^3+b^3 = (a+b)^3 – 3ab(a+b)$, and $a^3–b^3 = (a–b)^3 + 3ab(a–b)$ — this avoids separately computing $a^2 \mp ab+b^2$
Banker's Discount & True Discount	Present Worth (PW) is the sum which, invested today at the given rate, would grow into the Amount (A) due at the future date. True Discount (TD) = A – PW; Formula: PW = 100×A / (100 + R×T), so TD = A – PW, where R is the annual rate and T is the time in years; Banker's Discount (BD) is simpler but less accurate — it's just the simple interest on the full future Amount for the unexpired time: BD = A×R×T / 100. BD is always ≥ TD; Banker's Gain (BG) = BD – TD — the extra the banker effectively earns by charging interest on the full future amount instead of the smaller present worth; Useful shortcuts once you already have some of these values: Banker's Gain = TD^2 / PW = (BD × TD) / A. These let you find BG directly without separately working out R and T; Reverse problems that give you BD and TD (or BG) directly can be solved step by step: first find BG = BD – TD, then PW = TD^2/BG, then A = PW + TD, and finally use TD = A×RT/(100+RT) to recover the rate (for a given time) or the time (for a given rate)
Boat and Stream: downstream and upstream speed	Downstream = boat + stream; Upstream = boat – stream. Boat speed = (D+U)/2; Stream speed = (D–U)/2. Round-trip average speed = 2DU/(D+U). Time ratio upstream:downstream inverts to give the speed ratio. Stream speed must always be less than boat speed (S < B).
Boat and Stream Solved Questions	If a boat's speed in still water is B and the stream's speed is S: downstream speed (with the current) = B + S. Upstream speed (against the current) = B – S; Given the downstream speed (D) and upstream speed (U), you can recover the boat's and stream's own speeds by reversing the formula: boat speed = (D + U) / 2, stream speed = (D – U) / 2; Once you have a downstream or upstream speed, it's an ordinary Time, Speed and Distance problem — apply Time = Distance / Speed as usual; For a round trip covering the same distance downstream and back upstream, use the same two-unequal-speeds average-speed shortcut from Time, Speed and Distance: average speed = 2 × downstream speed × upstream speed / (downstream speed + upstream speed); If a question gives the TOTAL time for a round trip instead of one-way data, set up one equation: total time = distance/downstream speed + distance/upstream speed, and solve it for the unknown (usually the distance); If the time taken upstream is some multiple n of the time taken downstream for the same distance, then downstream speed = n × upstream speed (since time and speed are inversely related for a fixed distance) — this gives boat speed : stream speed = (n + 1) : (n – 1); Read carefully whether a stated "speed of the boat" means its still-water speed or an already-combined downstream/upstream speed — the wording decides which formula to apply first

Chain Rule (Compound Proportion)	Identify whether each pair of quantities is DIRECTLY proportional (both increase together, like cost and quantity) or INVERSELY proportional (one increases as the other decreases, like workers and days for a fixed job); More workers → fewer days needed (inverse). More hours per day → fewer days needed (inverse). More work to do → more days needed (direct); Set up the unknown as: new value = old value × (ratio that makes sense for each relationship) — for an inverse relationship, the ratio is old/new; for a direct relationship, it's new/old; Chain every relevant ratio together in one single equation rather than solving pair by pair — this avoids rounding errors and mixed-up intermediate steps; Food/ration-duration problems are an inverse-proportion chain rule in disguise: more people sharing a fixed stock of food means it lasts fewer days. If the headcount changes partway through, split into stages — work out how much stock is used before the change, how much is left, and divide that remainder by the new headcount; If the total amount of WORK itself also changes (not just workers/days/hours) — e.g. a longer wall, or a bigger task — add one more direct-proportion factor to the chain: (new work amount / old work amount), since more work needs more days, all else equal
Geometry: Circles — Concepts and Applications	Perpendicular from centre bisects a chord. Central angle = 2 × inscribed angle on the same arc. Opposite angles of a cyclic quadrilateral sum to 180°. Tangent ⊥ radius at the point of contact; two tangents from one external point are equal. Direct common tangent uses (R–r); transverse common tangent uses (R+r), only when circles are separate.
Geometry: Lines and Triangles — Concepts and Applications	Straight line angles sum to 180°; vertically opposite angles are equal; triangle angles sum to 180°; exterior angle = sum of remote interior angles. Similar triangles: sides scale linearly, areas scale by the square. Pythagorean converse: $a^2+b^2=c^2$ means the triangle is right-angled opposite c. Angle bisector theorem: BD:DC = AB:AC. Centroid divides each median 2:1 from the vertex.
Geometry: Lines, Angles & Triangles	The angles of any triangle always add up to 180°. Given any two angles, subtract their sum from 180° to find the third; When angles are given as a ratio (like 2:3:4), assign each part a variable (2x, 3x, 4x), set their sum equal to 180°, solve for x, then multiply back to get each actual angle; The exterior angle of a triangle (formed by extending one side) equals the SUM of the two interior angles that are not adjacent to it — this is faster than working out the third interior angle first and subtracting from 180°; An exterior angle and its adjacent interior angle always add up to 180° (they form a straight line) — useful for converting between the two when a question mixes them; Triangle inequality: the sum of any two sides of a triangle must be greater than the third side. Use this to check whether three given lengths can even form a triangle before doing any angle work; In an isosceles triangle, the two angles opposite the two equal sides are themselves equal; in an equilateral triangle, all three sides are equal and all three angles are 60°; When a transversal cuts two parallel lines: corresponding angles are equal, alternate angles are equal, and co-interior (allied) angles add up to 180°. These three facts are usually what let you find an unknown angle in a parallel-lines figure
Heights and Distances — Concepts and Applications	$\tan\theta$ = (vertical rise)/(horizontal distance), from the observer's eye level; add eye height if the ground-level height is asked for. $\tan30°=1/\sqrt{3}$, $\tan45°=1$, $\tan60°=\sqrt{3}$. Two-position problems: write one tan equation per position with the SAME unknown height, and use the known walking distance as the difference of the two horizontal distances. Ladder problems: height = length × $\sin\theta$ (ladder is the hypotenuse).

Height and Distance Solved Questions	The three standard angles and their tangent values (the ratio used most often here): tan 30° = 1/√3, tan 45° = 1, tan 60° = √3; Angle of elevation is measured UP from the horizontal (looking up at something); angle of depression is measured DOWN from the horizontal (looking down at something) — both are measured from the horizontal line, not from the ground or the object; tan(angle) = opposite side / adjacent side, where "opposite" is the height (vertical) and "adjacent" is the horizontal distance from the base — draw the triangle first, label height and distance, then apply tan; For a slanted line like a ladder or a wire (the hypotenuse), use sin or cos instead of tan: cos(angle) = adjacent/hypotenuse when the angle is with the ground, sin(angle) = opposite/hypotenuse; If the angle of elevation changes as an observer walks a known distance toward or away from the base, set up a separate tan equation at each position (both sharing the same unknown height), then solve the two equations together; For a "flagpole/antenna mounted on top of a building" question, where you're given the angles of elevation to the bottom and the top of the flagpole from the same point, draw two right triangles sharing the same horizontal distance, find each height, and subtract to get the flagpole's own length
LCM and HCF: prime factorization and the product shortcut	HCF = common prime factors at lowest power; LCM = all prime factors at highest power. HCF × LCM = a × b holds ONLY for two numbers, never three or more. Same-remainder problems: HCF of (numbers minus the remainder). Large numbers: use successive (Euclidean) division. Fractions: HCF = HCF(num)/LCM(den), LCM = LCM(num)/HCF(den). Co-prime numbers: LCM = their product.
LCM and HCF Solved Questions	HCF (Highest Common Factor) is the largest number that divides every given number exactly. LCM (Lowest Common Multiple) is the smallest number that every given number divides into exactly; Prime factorization method: break each number into prime factors. HCF = product of the common prime factors, each at its LOWEST power across the numbers. LCM = product of all prime factors (common and uncommon), each at its HIGHEST power; Useful shortcut for two numbers a and b: HCF(a, b) × LCM(a, b) = a × b. This lets you find one directly from the other without refactorizing. If a and b are co-prime (HCF = 1), this simplifies further: LCM = a × b directly; For "largest number that divides X, Y and Z leaving the same remainder R" questions, find the HCF of (X – R), (Y – R) and (Z – R) instead of X, Y, Z directly; "Smallest number divisible by a, b and c" = LCM(a, b, c). "Smallest number that leaves the SAME remainder R when divided by each of a, b and c" = LCM(a, b, c) + R; "Smallest number that, when divided by a, b and c, leaves remainders each exactly k less than their own divisor" (check this holds for every divisor first) = LCM(a, b, c) – k; Events that repeat at fixed intervals and need to coincide again (bells ringing together, runners starting together, lights changing together) is an LCM application: the time until they next coincide is the LCM of their individual intervals
HCF and LCM of Fractions Solved Questions	HCF of fractions = HCF of the numerators / LCM of the denominators — provided every fraction has first been written in its lowest terms (numerator and denominator sharing no common factor); LCM of fractions = LCM of the numerators / HCF of the denominators — under the same lowest-terms condition; Always reduce every given fraction to lowest terms FIRST, before taking the HCF of the numerators or the LCM of the denominators. Skipping this can silently give a wrong answer: for 2/4 and 3/6 (both equal to 1/2), the raw unreduced formula gives HCF(2,3)/LCM(4,6) = 1/12 — which is NOT the correct HCF, since 1/2 itself is a far larger value that both fractions are exact multiples of. Reducing first (1/2 and 1/2) correctly gives HCF = 1/2; Simplifying the FINAL answer (if it isn't already in lowest terms) is a separate, optional last step for a cleaner answer — it does not substitute for reducing the inputs first, and won't reliably fix an error introduced by skipping that step; Check your answer: an HCF of fractions must divide evenly into every given fraction (each original ÷ your HCF should be a whole number); an LCM of fractions must itself be evenly divisible by every given fraction (your LCM ÷ each original should be a whole number); The whole process mirrors whole-number HCF/LCM exactly — reduce each fraction, apply HCF/LCM to the numerators and denominators as described, then simplify the result if needed. The only new skill is handling a numerator and a denominator together instead of a single number
Mensuration: 2D — Concepts and Applications	Rectangle: area=lb, perimeter=2(l+b). Square: area=s^2, perimeter=4s. Triangle: area=½×base×height. Rhombus: area=½×d_1×d_2. Trapezium: area=½×(sum of parallel sides)×height. Circle: circumference=2πr, area=πr^2. Sector: both area and arc length use the SAME fraction (angle/360°) of the full circle's formula. Regular hexagon shortcut: area=(3√3/2)×$side^2$.

Mensuration (Area & Perimeter)	Perimeter is the distance around a shape; area is the space it covers. Rectangle: Perimeter = 2(length + breadth), Area = length × breadth. Square (a rectangle with equal sides): Perimeter = 4 × side, Area = side²; Circle: Circumference = 2πr, Area = πr². Use π ≈ 22/7 when the radius is a multiple of 7 (it cancels cleanly); otherwise use π ≈ 3.14; Triangle: Area = (1/2) × base × height. For a right-angled triangle specifically, the two legs meeting at the right angle can be used directly as base and height. When only the three sides a, b, c are known (no height), use Heron's formula instead: Area = √(s(s−a)(s−b)(s−c)), where s = (a+b+c)/2 is the semi-perimeter; Parallelogram: Area = base × height, where height is the perpendicular distance between the pair of parallel sides, not the length of the slanted side. Rhombus (a parallelogram with all sides equal): Area = (1/2) × d_1 × d_2, using the two diagonals; Trapezium: Area = (1/2) × (sum of the two parallel sides) × height; Equilateral triangle (a special case where Heron's formula simplifies): Area = (√3/4) × side²; Sector of a circle (a "slice" of angle θ degrees out of the full 360°): Area = (θ/360) × πr², and the arc length = (θ/360) × 2πr. A semicircle is just the θ = 180° case of both formulas; For "path around a field" or "border" questions, find the outer shape's area and the inner shape's area separately, then subtract — don't try to compute the path's area directly as one piece
Mensuration: 3D — Concepts and Applications	Cuboid: volume=lbh, TSA=2(lb+bh+hl). Cube: volume=s³, TSA=6s². Cylinder: volume=πr²h, CSA=2πrh, TSA=2πr(h+r)=CSA+2 circular ends. Cone: volume=πr²h/3, lateral area=πrl (l=slant height). Sphere: volume=4πr³/3, SA=4πr². Hemisphere: volume=(2/3)πr³, curved area=2πr², total area=3πr² - three distinct quantities. Melting/recasting conserves volume, not surface area.
Mixture and Alligation — Concepts and Applications	Mixture value/concentration = weighted total/total quantity; alligation ratio = (high − mean):(mean − low). Combining two solutions: add the actual ingredient amounts (volume × concentration) before dividing by total volume. Retained fraction after n replacement operations = (1 − removed/total)^n - a geometric decay, not a linear one.
Mixture and Alligation Solved Questions	Alligation rule: to mix a cheaper item (price C) and a dearer item (price D) so the mixture's mean price is M, the required ratio is cheaper : dearer = (D − M) : (M − C). This only makes sense when M is strictly between C and D — if the target mean is outside that range, the mixture as described is impossible; This is just the weighted-average equation rearranged — you can always sanity-check an answer by computing the weighted average directly from the ratio you found and confirming it equals the target mean; The same rule works for anything with a "concentration" — milk-water mixtures, alloy compositions, even mixing two classes' average marks — not just prices; For "cost price vs selling price" alligation questions (like diluting milk with free water to make a profit), treat water's cost as ₹0 and work out the mixture's effective cost price from the profit percentage first, before applying the ratio formula; For mixing three or more components at once, the two-term alligation shortcut does not directly apply — fall back to the weighted-average definition: total value of all components ÷ total quantity; Repeated dilution ("remove and replace"): if a container holds volume V of a pure liquid, and each time x units are withdrawn and replaced with water, then after n such operations the pure liquid remaining = V × $(1 - x/V)^n$. This is a geometric decay, not a linear one — don't just subtract nx
Partnership — Concepts and Applications	Profit-sharing weight = Σ(capital × duration); deduct stated salary/commission first, then share the remaining profit by these weights. A withdrawing/joining partner's weight only counts their actual invested months. Commission on TOTAL profit: compute directly (rate × total). Commission on profit AFTER commission: set up an equation, since the unknown appears on both sides.

Partnership Solved Questions	If all partners invest for the SAME length of time, profit is split simply in the ratio of their capital amounts; If partners invest for DIFFERENT durations, each partner's share of the ratio is capital × time — this product is what determines the split, not the capital amount on its own; This can produce a non-obvious ratio: a smaller capital invested for much longer can earn an equal or even larger share than a bigger capital invested briefly; If a partner changes their invested amount partway through the period (adding or withdrawing money), split their time into separate stretches and compute capital × time for each stretch, then add them up to get that partner's total "capital-months"; If a partner joins later than the others, their time factor is only the months their money was actually invested, not the full period — always base each partner's time on when THEIR capital entered the business; For a "working partner" who manages the business and earns an extra fixed fee for that on top of their investment return: subtract the fee from the total profit first, then split only the REMAINDER in the capital × time ratio; Reverse problems (given the profit-sharing ratio, find the investment ratio) just run the same rule backwards: capital × time ratio = profit ratio, so when the time periods are equal, the capital ratio equals the profit ratio directly
Permutation & Combination	Permutation counts ARRANGEMENTS, where order matters (ABC is different from BCA). Combination counts SELECTIONS, where order doesn't matter (choosing the group {A, B, C} is the same regardless of order); Factorial is the building block for both: $n! = n \times (n-1) \times (n-2) \times \ldots \times 1$, and by convention $0! = 1$; Permutation of n items taken r at a time: ${}^{n}P_{r} = n! / (n-r)!$. Combination of n items taken r at a time: ${}^{n}C_{r} = n! / (r! \times (n-r)!)$ — a combination is a permutation divided by the r! ways to arrange the chosen r items, since order no longer matters; Watch the wording: "arrange", "order", "ranks", "different ways to stand in a line" signal permutation. "Choose", "select", "form a committee" (where only the group matters, not who does what within it) signal combination; If a word has repeated letters, arranging all of them is NOT simply n! — divide by the factorial of each letter's repeat count: arrangements = n! / (p! × q! × …), where p, q, … are how many times each distinct letter repeats; Fundamental counting principle: for a task done in several independent steps, each with its own number of choices, multiply the choices together. This is what lets you build up passwords, codes or registration numbers digit by digit, not just count subsets; "At least" conditions (e.g. "a committee with at least 2 women") are usually solved by splitting into cases and adding (exactly 2, exactly 3, …), or by subtracting the complement (total ways minus the ways with fewer than the minimum) — whichever needs fewer cases; "Two particular people/items must always be together" in a row: glue them into a single block, arrange the reduced set of units, then multiply by the number of ways to arrange the people WITHIN that block (2! if it's a pair)
Circular Permutations Solved Questions	n distinct people around a circular table: (n−1)! arrangements, not n! — fixing one person's seat removes the rotational duplicates (rotating everyone by one seat is still counted as the same arrangement); For a necklace or bracelet (where flipping it over gives the same arrangement, not just rotating it), divide by 2 as well: (n−1)!/2; When two specific people must sit together, glue them into a single block first. Arrange the resulting (reduced count) of entities in a circle using (count−1)!, then multiply by 2 for the two ways the glued pair can face each other; For "must NOT sit together," find the total circular arrangements, subtract the "sit together" count — this is faster and less error-prone than trying to count the "not together" cases directly; The (n−1)! rule assumes only RELATIVE position matters (an ordinary round table with unmarked seats). If the seats themselves are distinguishable — numbered chairs, or fixed compass directions — then rotating the whole group DOES create a genuinely new arrangement, and the count is the ordinary n!, just like arranging people in a row; To choose r people out of n and then seat that group around a circular table, first choose the group with ${}^{n}C_{r}$, then arrange it circularly with (r−1)!: total = ${}^{n}C_{r} \times (r-1)!$
Problems on Ages: one variable, carried through	Assign x to one age, express all others in terms of x. 'After/ago N years' shifts EVERY age by N. A given ratio becomes one multiplier (3x, 5x), not two separate unknowns. The age DIFFERENCE between two people is constant over time - a powerful shortcut. Three-person chains: assign x to the person with the fewest multipliers. Two time conditions (past AND future) on the same pair need simultaneous equations.

Problems on Ages Solved Questions	Assign a variable to one unknown age (usually the one appearing in the simplest ratio or relationship), then express every other age in the problem in terms of that same variable; "A is twice as old as B" means A = 2B — translate sentences in the same subject-first order they're written, not reversed; For "after N years" or "N years ago" conditions, add or subtract N from EVERY person's age in the problem, including the one already defined in terms of x — forgetting to adjust one person's age is the most common mistake here; When a ratio is given directly (like "ages are in the ratio 3 : 5"), use a single multiplier — 3x and 5x — rather than two separate unknowns. This turns the problem into one variable, solvable with just one more equation from the rest of the problem; Useful check: the DIFFERENCE between any two people's ages never changes over time. If you compute it once from the present ages, it must still hold true at every other time mentioned in the question — a quick way to catch an arithmetic slip; When a problem gives two separate conditions at two different times with genuinely two unknowns (not a simple ratio to start with), set up two equations in the same two variables and solve them together by substitution or elimination; After solving, re-check against EVERY condition in the question, not just the last one you used to solve for x — a value that satisfies one condition but not another means an earlier step went wrong
Profit, Loss and Discount — Concepts and Applications	Profit/loss% uses cost price; discount% uses marked price; selling price = marked price × retained discount factors (multiply, don't add, successive discounts). Include all stated costs. False-weight dealers: profit% = (claimed−actual)/actual × 100. Recovering marked price from a discounted selling price: divide by the retained fraction, never add the discount % back.
Profit and Loss Solved Questions	Profit = Selling Price (SP) − Cost Price (CP), when SP > CP. Loss = CP − SP, when CP > SP; Profit% or Loss% is always calculated on the Cost Price unless a question explicitly says otherwise: (Profit or Loss / CP) × 100. A "profit margin on sales" (used in real business contexts) is calculated on SP instead — the two percentages are not the same number for the same rupee profit, so always check which base a question intends; Discount is calculated on the Marked Price (MP), not the cost price: SP = MP × (1 − Discount%/100). Mixing this up with CP is the most common mistake; When an item is marked up by a% and then discounted by b%, the net effect on profit is the same successive-change formula as percentages: net% = a + b + (ab/100). This also means a markup and a discount of the same size never cancel out — the result is always a net loss (see the worked example); False-weight or false-measure questions ("claims to sell at cost price but uses an understated weight") are profit questions in disguise: the dealer's gain% = (error / actual quantity given) × 100, where error = claimed quantity − actual quantity; A useful identity: if two items are each sold at the same selling price, one at a gain of x% and the other at a loss of x%, the overall result is always a loss of (x/10)% squared, i.e. (x^2/100)% — never a break-even, even though +x% and −x% look like they should cancel; Successive discounts (e.g. "20% off, then an extra 10% off") combine the same way as successive percentage changes, not by simply adding the discount percentages — see the dedicated Successive Discount lesson for more on this; Checks: a discount must bring SP below MP; a profit must bring SP above CP; profit% and loss% on the same CP are not interchangeable with profit% and loss% on different CPs without first converting to actual rupee amounts
Successive Discount Solved Questions	Apply each discount one at a time, in order, to whatever price is left after the previous discount — never add the percentages together and apply them once; The single discount equivalent to two successive discounts d1% and d2% is d1 + d2 − (d1 × d2)/100 — always a little LESS than the simple sum d1 + d2, because the second discount is applied to an already-reduced price; Successive discounts almost always leave the buyer paying MORE than a single discount equal to their sum would — "25% and 10% off" is a smaller saving than a flat "35% off," even though 25 + 10 = 35; Two successive 50% discounts do NOT make an item free — the second 50% is off the already-halved price, so the combined effective discount is 75%, not 100%; For THREE successive discounts, combine them two at a time using the same formula repeatedly: first combine d1 and d2 into one equivalent discount, then combine that result with d3 — this is easier than tracking three percentages in your head at once; Reverse problems (given the final price, or the overall equivalent discount, find one individual discount) just run the multiplication backwards: divide the overall multiplier by the known discount's multiplier to isolate the unknown one

BODMAS, Fractions, Decimals, Roots and Indices	Brackets → powers/roots → × and ÷ left to right → + and – left to right; fraction division uses reciprocals; recurring tails cancel by subtraction; compare ad and bc; principal √ is non-negative; add/subtract/multiply exponents for product/quotient/power of a power.
Simplification (BODMAS) Solved Questions	BODMAS gives the order of operations: Brackets, Of, Division, Multiplication, Addition, Subtraction — solve innermost brackets first, not left to right across the whole expression; "Of" means multiply, but by the standard teaching convention for these exams, every "of" in the expression is resolved before any separate division or multiplication is carried out — this is the step that trips people up when they skip straight to left-to-right; Division and multiplication rank EQUALLY with each other and are done left to right in the order they appear — it's not "always divide before multiplying". The same is true for addition and subtraction; Work brackets from the innermost outward: solve ( ) first, then { }, then [ ], substituting each result back in before moving to the next layer out; A vinculum (a bar drawn over part of an expression) has the highest priority of all — even higher than brackets. Simplify everything under the bar first, as its own self-contained BODMAS calculation, before touching anything outside it; When an expression mixes fractions and decimals, convert everything to one form (usually fractions, since they combine exactly) before calculating — combining 0.6 and 3/5 without converting one of them is a common source of arithmetic slips; Check your work by re-simplifying just the innermost piece on its own and confirming it matches what you substituted back in — this catches most "wrong order" mistakes before they compound through the rest of the expression
Approximation Solved Questions	Round every number in the expression to a convenient nearby value FIRST — nearest whole number for decimals, nearest multiple of 10/100 for percentages and large numbers, nearest perfect square under a square root; Then calculate normally using BODMAS with the rounded numbers — the goal is a value close to the exact answer, not the exact answer itself; Round consistently: if you round one number up, the overall error is usually small as long as you don't round several numbers in the same direction by a large margin; This is a pure speed technique for multiple-choice questions where the options are spread far enough apart that an approximate value clearly identifies the right one; For squares, cubes and roots, round to the nearest number whose square, cube or root is easy to compute by hand — e.g. $28.9^2 \approx 29^2 = 841$, or the cube root of 215.5 ≈ cube root of 216 (a perfect cube) = 6; In a PRODUCT or DIVISION of several rounded numbers, rounding more than one number in the SAME direction compounds the error (both errors multiply together); rounding some up and some down tends to partially cancel it out; In a SUM or DIFFERENCE of several terms, rounding errors simply add (or partially cancel if some terms round up and others round down) — this is generally a smaller risk than in multiplication, where the errors combine multiplicatively rather than just adding
Stocks and Shares Solved Questions	Face Value (FV) is the share's printed/nominal value (commonly ₹100 unless stated otherwise) — dividend is always calculated as a percentage of FV, never of the market price; Market Value (MV) is the price actually paid to buy the share. A share "at a premium of ₹X" means MV = FV + X; "at a discount of ₹X" means MV = FV – X; Number of shares bought = Investment ÷ MV. Annual income = Number of shares × Dividend per share, where dividend per share = (Dividend % × FV) / 100; Rate of return (yield) is the dividend as a percentage of what was actually paid: Yield% = (Dividend per share / MV) × 100 — this is what lets you compare returns across shares bought at different prices; Stock is often quoted differently in exam wording: "r% ₨N stock at M" means a holding with total FACE VALUE ₨N, paying r% dividend on that face value, currently trading at market price M (per ₨100 of face value). Cost to buy this holding = N × M/100. Annual income = r% of N (the face value) — not of the cost paid; To decide which of two stocks is the better investment, compare their yields (return per rupee actually invested): Yield% = (Dividend% × FV) / MV. The higher yield is the better buy, even if its advertised dividend percentage looks smaller
Trigonometry — Concepts and Applications	$\sin\theta=\text{opp/hyp}$, $\cos\theta=\text{adj/hyp}$, $\tan\theta=\text{opp/adj}$; cosec, sec, cot are their reciprocals. $\sin^2\theta+\cos^2\theta=1$; $\sec^2\theta-\tan^2\theta=1$; $\text{cosec}^2\theta-\cot^2\theta=1$. Complementary angles swap sin↔cos, tan↔cot, sec↔cosec. Any two known sides of a right triangle give all six ratios directly, without needing the angle itself. Standard values at 0°,30°,45°,60°,90°: sin = 0, 1/2, 1/√2, √3/2, 1 (cos reversed).

Trigonometric Ratios & Identities	The core identity: $\sin^2\theta + \cos^2\theta = 1$, for any angle θ. Given one of sin θ or cos θ, this gives the other directly; Two derived identities, both built from the core one: $1 + \tan^2\theta = \sec^2\theta$, and $1 + \cot^2\theta = \text{cosec}^2\theta$; Standard angle values worth knowing by heart: sin 30° = cos 60° = 1/2; sin 60° = cos 30° = √3/2; sin 45° = cos 45° = 1/√2; tan 45° = 1; tan 30° = 1/√3; tan 60° = √3; For an acute angle, all ratios are positive, so taking the square root in an identity only ever needs the positive root — this simplifies working with $\sin^2\theta + \cos^2\theta = 1$ in practice; Reciprocal relationships: cosec θ = 1/sin θ, sec θ = 1/cos θ, cot θ = 1/tan θ — convert between a ratio and its reciprocal this way rather than re-deriving a triangle each time; Complementary angle relationships: sin(90° − θ) = cos θ, cos(90° − θ) = sin θ, and tan(90° − θ) = cot θ — useful for simplifying expressions that mix an angle with its complement; Edge values worth knowing: sin 0° = 0, cos 0° = 1, sin 90° = 1, cos 90° = 0. tan 90° and cot 0° are undefined (they involve dividing by zero), which is why some identities only hold for angles strictly between 0° and 90°

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